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Effective Charge Influence on Form Factors Calculations of Some Sn Isotopes

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Abstract. *The longitudinal form factors of 102,104,106Sn nuclei were calculated for two cases: with and without the core polarization effects. In this study, different values of effective charge were used to calculate the longitudinal form factors. e_p=1.5e,e_n=0.88e,e_p=0.98e,e_n=0.88e ,e_p=0.5e ,e_n=0.4e and e_p=0.35e ,e_n=0.35e and are plotted using black, blue, red and green curves, respectively, in all figures. The form factors of isotopes were also studied within shell model, using 100Sn as the inert core, and the others particles distributed on (1H9/2, 2F7/2, 2F5/2, 3P3/2, 3P1/2, and 1I13/2), which constitute the model space. The calculations in this study are based on a harmonic oscillator potential with a constant magnitude parameter (b) using the first-order correction of the Cohen-Korath (CK) perturbation and interaction theory. The inclusion of core polarization effect enhances the diffraction pattern of longitudinal form factors and it has the similar behavior for all different values of effective charge.*

Key words: *Form Factors; Electron Scattering; Effective Charge; Core polarization.*

1. Introduction

Shell model calculations, within which the model space is performed and in which the nucleon is restricted to occupying a few orbitals, are unable to reproduce the measured constant moments or forces for translation without scale factors [1]. The shell model within a restricted model space is one of the models that has successfully described the static properties of nuclei when using effective charges [2][3]. Using the wave function in the model space for calculations of form factors alone is insufficient to reproduce electron scattering data[4]. There are model space effects that must be included, called fundamental polarization effects, in the calculations[5].

Various theoretical methods, such as the Hartree-Fock method with the effective harmonic oscillator interaction, the theory of finite Fermi systems, and the single-particle potential method, are used to calculate charge density distributions[6]. Nuclear structure models are also carefully tested, and comparisons among different cases of theoretical longitudinal electron scattering form factors[7].Interactions occur between the stable core and the valence nucleons. This interaction may cause a "polarization," or distortion, of the charge or mass distribution within the core[2], [8]. This distortion is not a permanent change in the core itself; it affects the properties and interactions of the valence nucleons, resulting in a dynamical response (a perturbative effect)[9].In general, "core polarization effects" describe how a stable internal structure (the core) responds to interactions with external particles or forces, and how this response, in turn, affects the properties and behavior of the external particles[10]. Essential for a thorough understanding of complex quantum systems, they are often first-order effects in perturbation theory[11].

2. Theory

The differential scattering cross section for scattering of an electron at a solid angle dΩ of mass M and a nucleus of charge Ze and in the plane wave Born approximation (PWBA) using the following equation[12]:

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{Mott} f_{\text{rec}} \sum_J \left|F_J(q)\right|^2 \qquad (1)$$

For high-energy electron scattering from a spinless nucleus, is given by[13]:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott}=\left[\frac{Z\alpha\cos(\theta/2)}{2E_i\sin^2(\theta/2)}\right]^2 \qquad (2)$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant which is representing the order of interaction, Z is the atomic number of the target nucleus, θ is the scattering angle and E_i is the energy of incident electron.The recoil factor of the nucleus is given by[6], [14]:

$$f_{rec}=\left[1+\frac{2E_i}{M}\sin^2(\theta/2)\right]^{-1} \qquad (3)$$

when M is the mass of the target.

For elastic scattering or inelastic scattering between an initial state (i) and final state for a known multipolarity J, as a function of momentum transfer (q) and scattering angle (θ), the total nuclear form factor $F_J(q,\theta)$ containing two parts that is the longitudinal part and the transverse part are given by[15]:

$$\left|F_J(q)\right|^2=\left(\frac{q_\mu}{q}\right)^4\left|F_J^{Long}(q)\right|^2+\left[\frac{q_\mu^2}{2q^2}+\tan^2(\theta/2)\right]\left|F_J^{Trans}(q)\right|^2 \qquad (4)$$

q_μ : four-momentum transfer is given by, (with $\hbar=c=1$ for abbreviation):

$$q_\mu^2 = q^2-(E_i-E_f)^2 \qquad (5)$$

with

$$q^2 = 4E_iE_f\sin^2(\theta/2)+(E_i-E_f)^2 = 4E_iE_f\sin^2(\theta/2)+\omega^2 \qquad (6)$$

$$\omega^2 = (E_i-E_f)^2 \qquad (7)$$

where E_i and E_f are the total energy of the electron before and after scattering, respectively. The quadratic transverse form factor, which represents the sum of the quadratic electric and magnetic form factors, is given by the equation[7][16]:

$$\left|F_J^T(q)\right|^2=\left|F_J^{El}(q)\right|^2+\left|F_J^{mag}(q)\right|^2 \qquad (8)$$

The form factor of a given multipolarity J as a function of momentum transfer q in term of reduced matrix elements of the transition operator is given as[17]:

$$\left|F_J(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\left\langle J_f \left\| \hat{T}_J(q) \right\| J_i \right\rangle\right|^2 \qquad (9)$$

where J_i and J_f are the total angular momentum of the initial and the final states, respectively, and $\hat{T}^{\eta}_{J}(q)$ is the electron scattering multipole operator.

The distribution of the nucleus charge with electron interaction leads to longitudinal or Coulomb scattering. The longitudinal form factor is related to charge density distributions (CDD) through the elements of the matrix[18].

Angular momentum (J) and momentum transfer (q) are the longitudinal and transverse form factors of electron scattering, which are calculated between the initial and final spin and isotropic states of the nuclear shell model, including these corrections[19]. Longitudinal and transverse form factors of electron scattering [20]:

$$\left|F^{\eta}_{J}(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\sum_{T=0,1}(-1)^{T_f - T_{f_z}} \frac{nn}{nn} \begin{pmatrix} T_f & T & T_i \\ -T_{f_z} & M_T & T_{i_z} \end{pmatrix} \left\langle J_f T_f \left\| \hat{T}^{\eta}_{\mathrm{B}\delta} \right\| J_i T_i \right\rangle\right|^2 \qquad (10)$$

where Z is the atomic number of the nucleus, and $F_{cm}(q)$ is the center of mass correction, which removes the spurious case caused by the movement of the center of mass when using the wave function of the shell model, when using the equation[21].

$$F_{cm(q)} = e^{q^2b^2/4\pi} \qquad (11)$$

where b is the harmonic oscillator size parameter or A is the nuclear mass number. The function $F_{fs}(q)$ is the free nucleon form factor, assumed to be the same for neutrons and protons, and takes the form[22].

$$F_{fs(q)} = \left[1 + \left(\frac{q}{4.33}\right)^2\right]^{-2} \qquad (12)$$

The effective momentum transfer is represented using the equation:

$$q_{eff} = q\left[1 + \frac{3ze^2}{2E_iR_c}\right] \qquad (13)$$

The reduced matrix elements of the electron scattering operator have two parts, one for the "fundamental polarization" matrix elements, and the other for the "model space" matrix elements[23].

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle = \left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP} \qquad (14)$$

where, $\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS}$ are the model-space reduced matrix elements ,

and, $\left\langle \Gamma_f \left\| \delta T^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP}$ are the core-polarization reduced matrix elements.

$\left|\Gamma_i\right\rangle$ and $\left|\Gamma_f\right\rangle$ are the model-space wave functions.

The model-space in the light Sn-shell nuclei is defined by the following orbits: ($1H_{9/2}$, $2F_{7/2}$, $2F_{5/2}$, $3P_{3/2}$, $3P_{1/2}$ and $1I_{13/2}$) .

The isotropic spin of the longitudinal operator and the reduced matrix elements in the spin-spin space between the initial and final particle states of the system, when the configuration is mixed, is given

in terms of the OBDM elements multiplied by the single-particle matrix elements of the longitudinal operator[24].

$$\langle \Gamma_f \| T^{\eta}_{\Lambda} \| \Gamma_i \rangle_p = \sum_{\alpha,\beta} OBDM(\Gamma_i, \Gamma_f, \alpha, \beta) \langle \alpha \| T^{\eta}_{\Lambda} \| \beta \rangle \quad (15)$$

where α and β represent the final and initial states in the single particle respectively, for the shell space model (jj56pn), $1G_{7/2}$,$2D_{5/2}$,$2D_{3/2}$,$3S_{1/2}$,$1H_{11/2}$. The elements of the (core - polarization) matrix can be written[25]:

$$\langle \Gamma_f \| \delta T^{\eta}_{\Lambda} \| \Gamma_i \rangle_{cp} = \sum_{\alpha,\beta} OBDM(\alpha, \beta, \Gamma_f, \Gamma_i, \Lambda) \langle \alpha \| \delta T^{\eta}_{\Lambda} \| \beta \rangle \quad (16)$$

The OBDM contains all the information about transitions of given multiplicities, which are imbedded in the model wave functions. The $OBDM$ element is calculated in terms of the isospin-reduced matrix elements[26].

$$OBDM(\tau_Z) = (-1)^{T_f - T_Z} \begin{pmatrix} T_f & 0 & T_i \\ -T_Z & 0 & T_Z \end{pmatrix} \sqrt{2}\, \frac{OBDM(\Delta T=0)}{2}$$

$$+\tau_Z (-1)^{T_f - T_Z} \begin{pmatrix} T_f & 1 & T_i \\ -T_Z & 0 & T_Z \end{pmatrix} \sqrt{6}\, \frac{OBDM(\Delta T=1)}{2} \quad (17)$$

The $OBDM(\Delta T)$ is defined [27]as

$$OBDM(i, f, j, \acute{j}, \Delta T) = \frac{\left\langle f \left\| \left[a^{\dagger}_{j} \otimes \tilde{a}_{\acute{j}} \right]^{J, \Delta T} \right\| i \right\rangle}{\sqrt{2J+1}\sqrt{2\Delta T+1}} \quad (18)$$

The operator $(\tilde{a}_{\acute{j}})$ annihilities a neutron or proton in the single nucleon state $(\acute{j})$ or the operator $(a^{\dagger}_{j})$ create a neutron or proton in the single nucleon state (j). First-order perturbation theory is used to define the OBDM used in the present work, the (core- polarization) of a single-particle nucleus is given by the equation[28].

$$\langle \alpha | \delta \hat{T}^{\eta}_{J} | \beta \rangle = \left\langle \alpha \left| V_{res} \frac{Q}{E - H^{(0)}} \hat{T}^{\eta}_{J} \right| \beta \right\rangle + \left\langle \alpha \left| \hat{T}^{\eta}_{J} \frac{Q}{E - H^{(0)}} V_{res} \right| \beta \right\rangle \quad (19)$$

Where the operator Q is the projection operator onto the space outside the model space or V_{res} is a residual nucleon-nucleon interaction.

Effective charge is the presence of a number of nucleons (protons and neutrons) within the nucleus. They do not behave as completely free particles carrying their "bare" or fundamental charge. The core of the nucleus (the Interior nucleus) and the nucleons interact with each other. This interaction leads to a phenomenon called (core polarization) . The account for the complex interaction effects between nucleons within the nucleus, the effective charge is a way to calculate this, When using the equation[29].

$$e_p = \left(1 + \frac{z}{A}\right) e \quad (20)$$

$$e_v = (2 \cdot 1) \left(\frac{Z}{A}\right) e \quad (21)$$

e_p: Effective charge of a proton and e_v: Effective charge of a neutron

Clark and colleagues the set of operators to derive an explicit form for the elastic charge form factor, cut off at the two-body terms, where the harmonic oscillator and the correlation parameter depend on each other through correlation function. The bulk parameter b was chosen to reproduce the measured ground state root mean square charge radius for these nuclei, as in the following equation[30]:

$$b = 1.005 \sqrt[6]{A} \quad (22)$$

This work aims to study the inelastic longitudinal form factors C0 for ^{102,104,106}Sn nuclei. It has been known that using the multi-particle shell model space to calculate the form factors alone is insufficient to describe the electron scattering data. Therefore, it is necessary to account for the (core-polarization) to include the effects of the model space[25].

The effective interaction (kh5082) were used to calculate the body density matrix element (OBDM) used in the present work, to generate the wave functions for a given transition in known nuclei using a modified version of the OXBASH envelope model code[31].

3. Results and Discussions:

3.1. Introduction

The longitudinal form factors of ^{102,104,106}Sn nuclei are calculated for two cases. The first is without core polarization effects and the second case uses core-shell polarization with MSDI (as a residual interaction), allowing particle and hole excitations. Excitations from space-dip orbitals (jj56pn) to higher orbitals allowed by first-order perturbation theory are also considered using the with and without core polarization effects. In this study different values of effective charge are performed in the calculation of longitudinal form factors .The effective charges values are: $e_p = 1.5e, e_n = 0.88e$,$e_p = 0.98e, e_n = 0.88e$, $e_p = 0.5e$, $e_n = 0.4e$ and $e_p = 0.35e$, $e_n = 0.35e$ and plotted with black, blue, red and green curve respectively in all figures.

3.2. The ^{102}Sn isotope (J^π T=0^+ 1)

The ground state of ^{102}Sn isotope is specified by $J^\pi T$= (0^+ 1) and has the configuration ($1H_{9/2}$, $2F_{7/2}$, $2F_{5/2}$, $3P_{3/2}$, $3P_{1/2}$ and $1I_{13/2}$) as inert core and (^{100}Sn) configuration as model space when two nucleons outside the core. The C0 elastic longitudinal form factors are calculated with size parameter b = 2.17. In these calculations, the effective interaction (kh5082) are used.

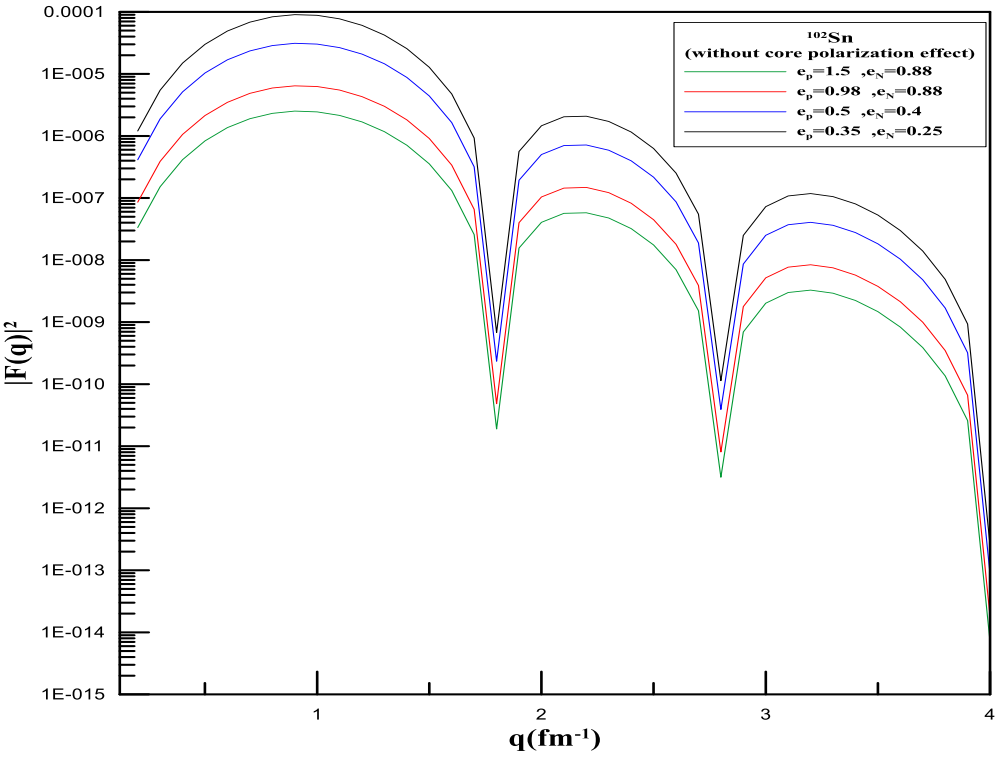

Figure (1): The elastic longitudinal form factor for ^{102}Sn without core polarization effect

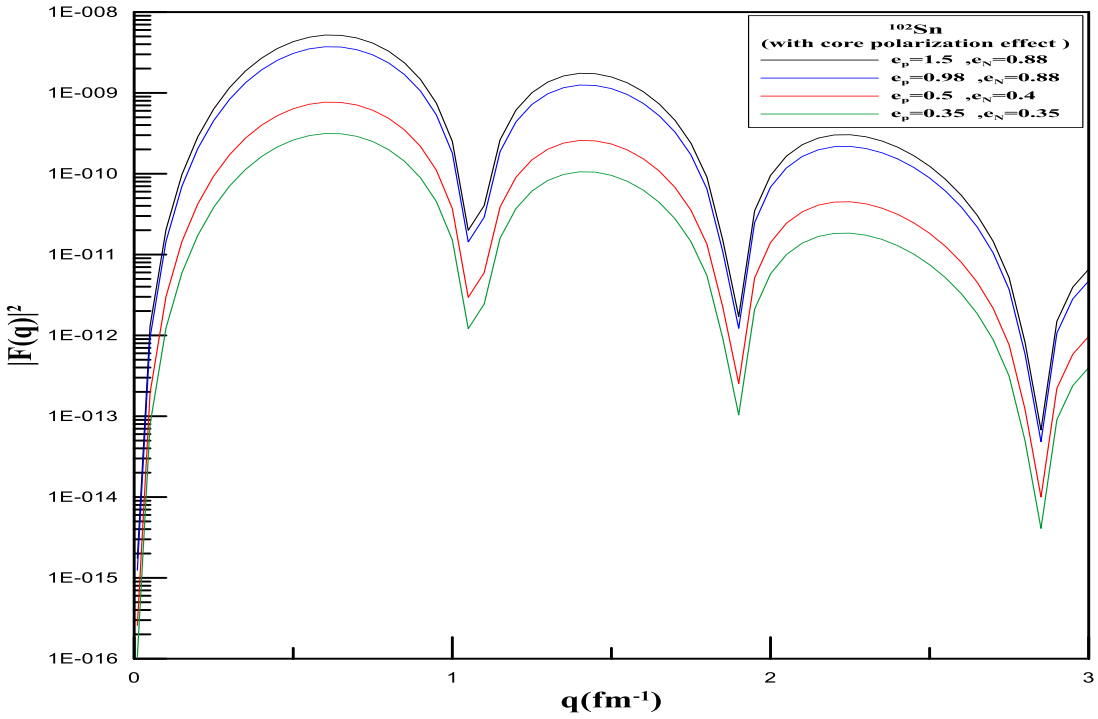

Figure (2): The elastic longitudinal form factor for ^{102}Sn with core polarization effect

As shown in figure1, the C0 calculations without core polarization effects has two diffraction minima and three diffraction maxima. The form factors with the effective charge (e_p = 1.5e, e_n = 0.88e) are dominated in all regions of momentum transfer as shown with black curve.

There is slightly difference in the longitudinal form factors between (e_p = 1.5e, e_n = 0.88e) and (e_p = 0.98e, e_n = 0.4e), but both has the same point of momentum transfer in the two minima $q = 0.72\ fm^{-1}, q = 1.3\ fm^{-1}$ and three maxima ($q = 0.5\ fm^{-1}, q = 1\ fm^{-1}$ and $q = 2\ fm^{-1}$) diffraction.

The form factors with effective charge (e_p = 0.5e, e_n = 0.4e) has significant effect where the form factor dominates in the high region of momentum transfer as shown in figure1 (red curve). Same effect appears in the form factors with effective charge (e_p = 0.35e, e_n = 0.35e).

Where in the last two cases the longitudinal form factor dominated in the large momentum transfer (third maxima) where q > 1.5 fm^{-1}. Also there are different in the two diffraction minima between form factors between the form factors with effective charge (e_p = 0.35e, e_n = 0.35e) and form factors with effective charge (e_p = 0.5e, e_n = 0.4e). Where the first minima and second minima (e_p = 0.35e, e_n = 0.35e) located at q = 1.4 fm^{-1} (as shown in figure1 green curve), and q = 1.6 fm^{-1} respectively. The first and second maxima of form factor with effective charge (e_p = 0.5e, e_n = 0.4e) has the same height, but the third maxima is higher than the first and second maxima as shown in figure1. The form factors with effective charge (e_p = 0.5e, e_n = 0.4e) has high peak in the third maxima and the first maxima is the lowest.

The inclusion of core polarization effects enhances the form factor calculations where each case has three maxima and three minima in the region from 0 to 3 fm^{-1}. Both the second and third maxima of the longitudinal form factor are each peak is smooth and symmetric higher than the first maxima when the core polarization is included. As shown in the figure, the effective charge increases, the form factor increases as the effective charge increases.

In each case, the height of the first maxima of from factor without core polarization effect(Figure1) but the inclusion of core polarization effects allows all those without core polarization effects in the second and third maxima. In addition, the peaks are wider than those of without core polarization effects.

3.3. The ^{104}Sn isotope (J^{π} T=0^{+} 2)

There are four nucleons outside the inert core. The longitudinal form factors without core polarization effects, has three diffraction maxima and two difference minima figure1. In all different values of effective charge, the longitudinal form factors dominated in the region of momentum transfer bounded from 1.7 fm^{-1} to 3 fm^{-1} which from second and third maxima.

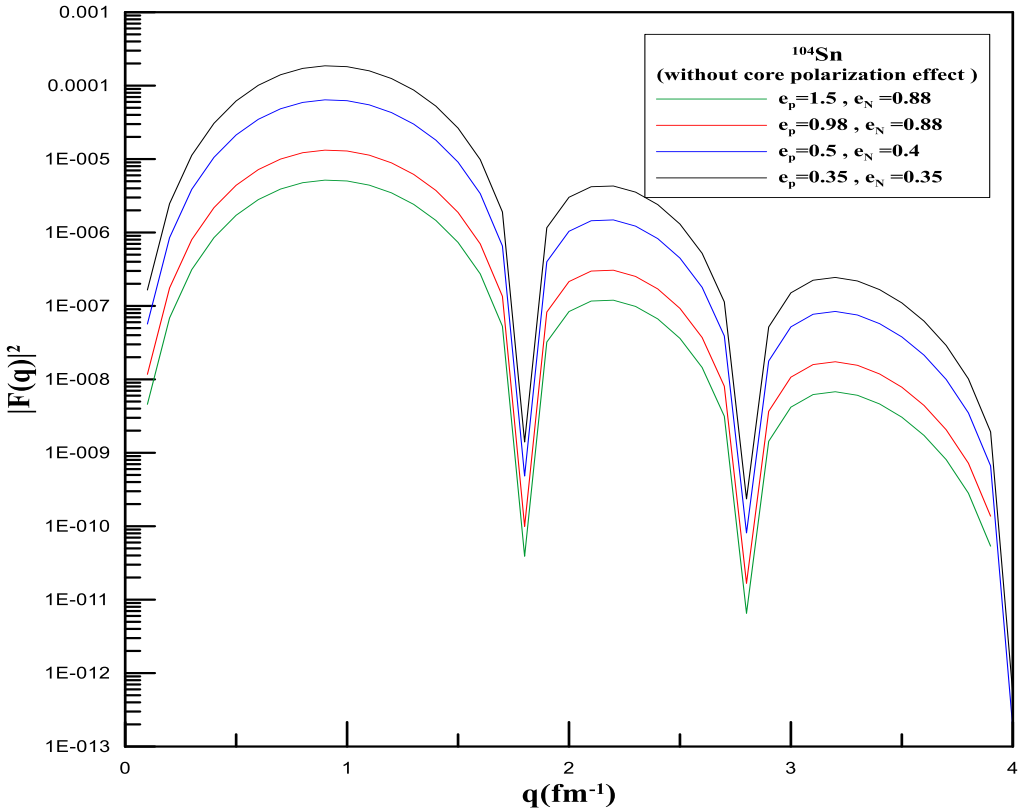

Figure (3): The elastic longitudinal form factor for ^{104}Sn without core polarization effect

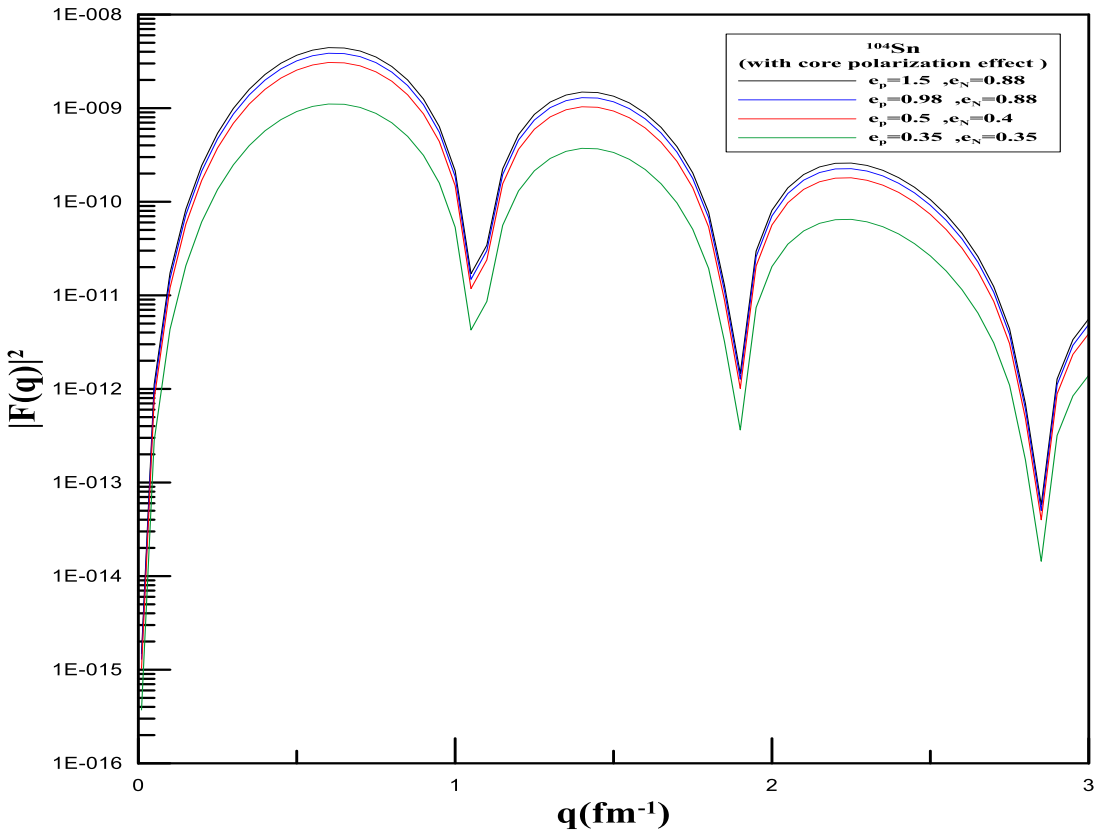

Figure (4): The elastic longitudinal form factor for ^{104}Sn with core polarization effect

As shown in the results explained in figure 3, the form factors with effective charge (e_p=0.35e, e_n=0.35e) is coincidence with the longitudinal form factors with the values of effective charge

(e_p=0.5e, e_n=0.4e). This is due to the low difference between the values of effective charge and there are four nucleons outside the core.

In all cases, the longitudinal form factors are considered in the third maxima and slightly different in the second minima, but in the first maxima, the difference is clearly where the case of longitudinal form factors with effective charge values (e_p=1.5e, e_n=0.88e) is highest. Also, in all cases the first and second diffraction minima are coincidence where located at $q = 0.7\ fm^{-1}$ and $q = 1.8\ fm^{-1}$, respectively .

The inclusion of core polarization effects are plotted in Figure(4). As shown in Figure the longitudinal form factors dominated as effective charge values increases. When core polarization effects take in to account, three diffraction maxima and three diffraction minima appears. In all cases of this one the peaks of each maxima has the same wide and same maxima location at q ,where the first, second and third maxima location at $= 0.5\ fm^{-1}$, $q = 1.5\ fm^{-1}$ and $q = 2.5\ fm^{-1}$, respectively, but it differ in the ligheit due to the different in the values of effective charge, where as the effective charge increase the height of peak increase . Also, the form factors dominant at low momentum transfer for $q = 0.01\ fm^{-1}$. The inclusion of core polarization effect in the calculation of form factors gives wide and smooth diffraction pattern.

3.4 ^{106}Sn isotope (J^{π} T= 0^{+} 3)

This isotope has six nucleons outside the inert core the ground state of ^{106}Sn is specified by (J^{π} T= 0^{+} 3) with excited energy =13.260Mev .

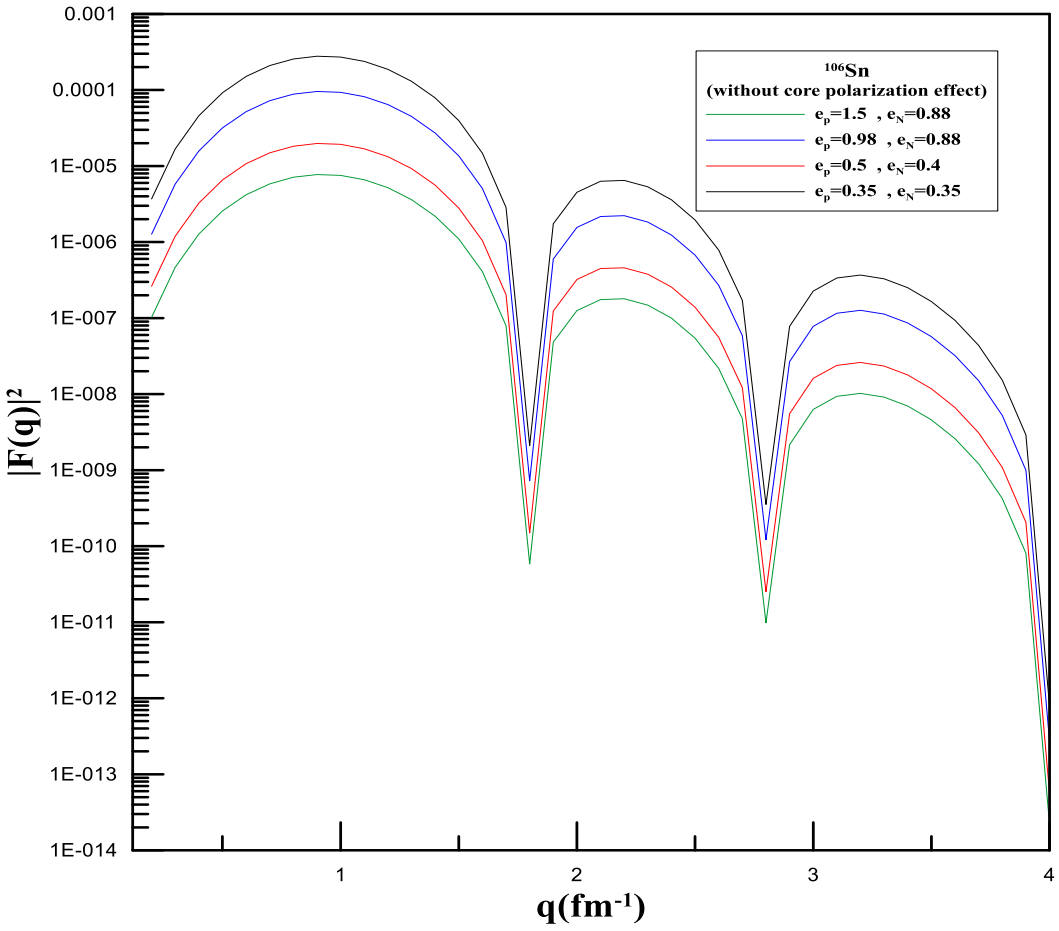

Figure (5): The elastic longitudinal form factors for ^{106}Sn without core polarization effect

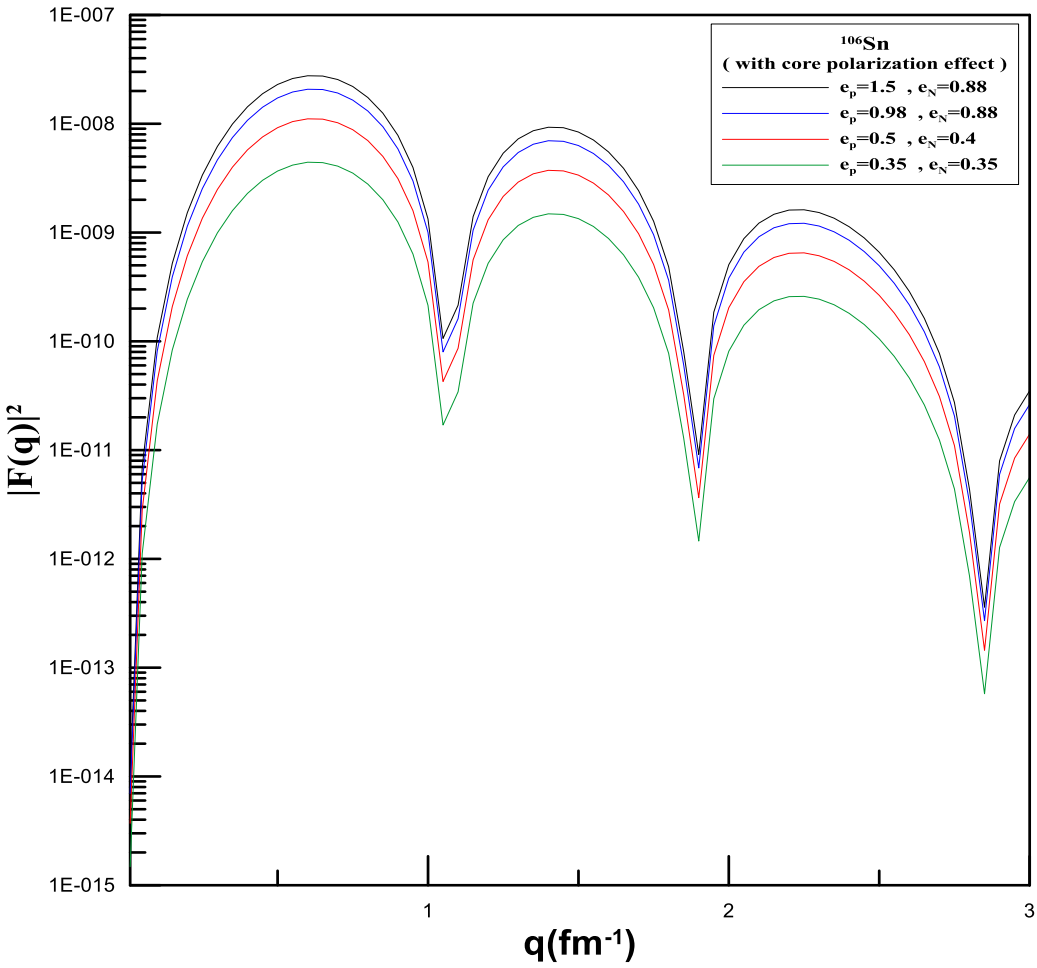

Figure (6): The elastic longitudinal form factors for ^{106}Sn with core polarization effect

Figure (5) shows the calculation of longitudinal form factors without core polarization effect with different values of effective charge. Each case has three diffraction maxima and two diffraction maxima except longitudinal form factors with effective charge values ($e_p = 1.5e\,, e_v = 0.88e$), where it has two diffraction maxima. The center of peak of first maxima for longitudinal form factors are coincidences in the two cases of effective charge values ($e_p = 0.35e\,, e_v = 0.35e$)($e_p = 0.5e\,, e_v = 0.4e$) where located at $q = 0.4fm^{-1}$and slightly different with the case of effective charge value ($e_p = 0.98e\,, e_v = 0.88e$) but there are wide different with these of effective charge value ($e_p = 1.5e\,, e_v = 0.88e$). The height of form factors is widely different for all cases at first maxima, but slightly different at third maxima. As shown from the results in figure(5) the first minima of form factors with values of effective charge ($e_p = 0.5e\,, e_v = 0.4e$) (red curve) a deep dip . There are deviated in the longitudinal form factors with effective charge ($e_p = 1.5e\,, e_v = 0.88e$) (black curve) in comparison with the others . In cases of effective charge($e_p = 0.98e\,, e_v = 0.88e$) and ($e_p = 1.5e\,, e_v = 0.88e$) the form factors are dominant at light q region (third maxima) as shown in Figure (5) with blue and black but in the case of effective charge ($e_p = 0.35e\,, e_v = 0.35e$) the form factors are dominant at low q region (first maxima) as shown in green curve. In the case of charge values)($e_p = 0.5e\,, e_v = 0.4e$) , the form factors are dominant in all q region .when the core polarization effect included ,the diffraction pattern enhanced ,where it become wide and smooth.

As shown in Figure (6) all different values of effective charge, the peaks have same center of maximum peak at q =0.5, 1.5, 2.3fm^{-1}, but they are different in height

The first (second and third) maxima located of $q = 0.5\ fm^{-1}$, $q = 1.5\ fm^{-1}$ and $q = 0.5\ fm^{-1}$ $q = 2.5\ fm^{-1}$, respectively .

4. Conclusion

The inclusion of core polarization effect enhances the diffraction pattern of longitudinal form factors and it has the similar behavior for all different values of effective charge. The calculations of longitudinal form factors without core polarization effects gives different behavior and deformed the diffraction pattern. The longitudinal form factors increase as effective charge increase. In the inclusion of core polarization effects the form factors dominant at low q region and the first, second and third maxima of each case alone has the same maxima peak location at q=0.5, 1.5, 2.3 fm^{-1}, but form factors without core polarization effects for each case alone has different location of maxima and minima along the momentum transfer ranges.

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