AJLLLSE GLOBAL RESEARCH NETWORK

AMERICAN Journal of Language, Literacy and Learning in STEM Education
Volume 3, Issue 6, 2025 ISSN (E): 2993-2769

Exploring Parabolas, Ellipses, and Hyperbolas in Space and their Applications in Various Sciences

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Abstract. *This work focuses on a detailed study of the properties of conic sections, namely parabolas, ellipses, and hyperbolas, as well as an analysis of their applications in various sciences.*

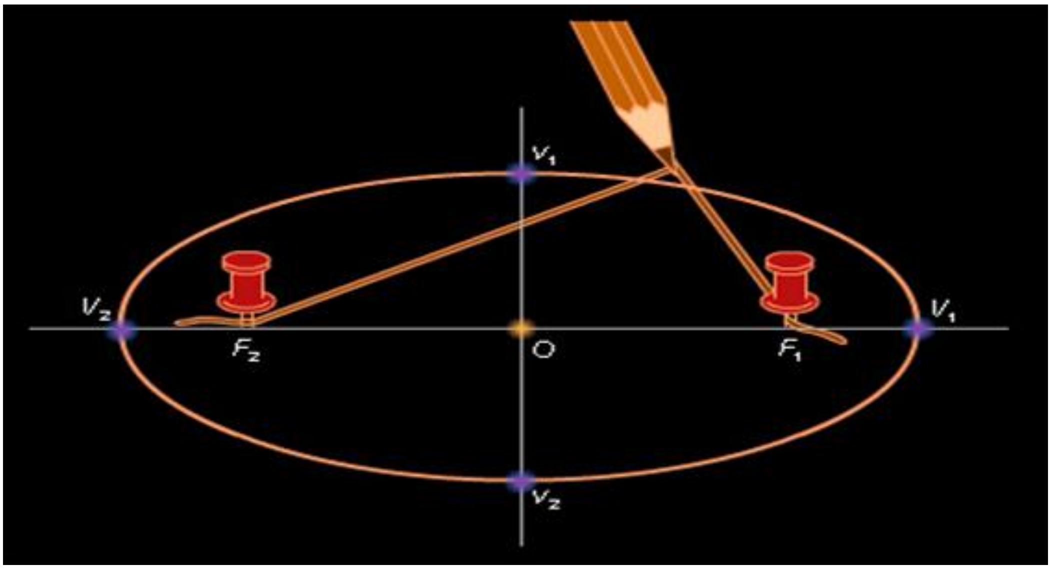
Key words: *Conic sections, parabola, ellipse, hyperbola, cylindrical surface.*

Studying conic sections as intersections of planes and cones, ancient Greek mathematicians also considered them as trajectories of points on a plane. It was established that an ellipse is the locus of points where the sum of the distances to two given points is constant; a parabola is the locus of points equidistant from a given point and a given line; a hyperbola is the locus of points where the difference of the distances to two given points is constant.

These definitions of conic sections as plane curves also suggest a method for constructing them using a stretched string.

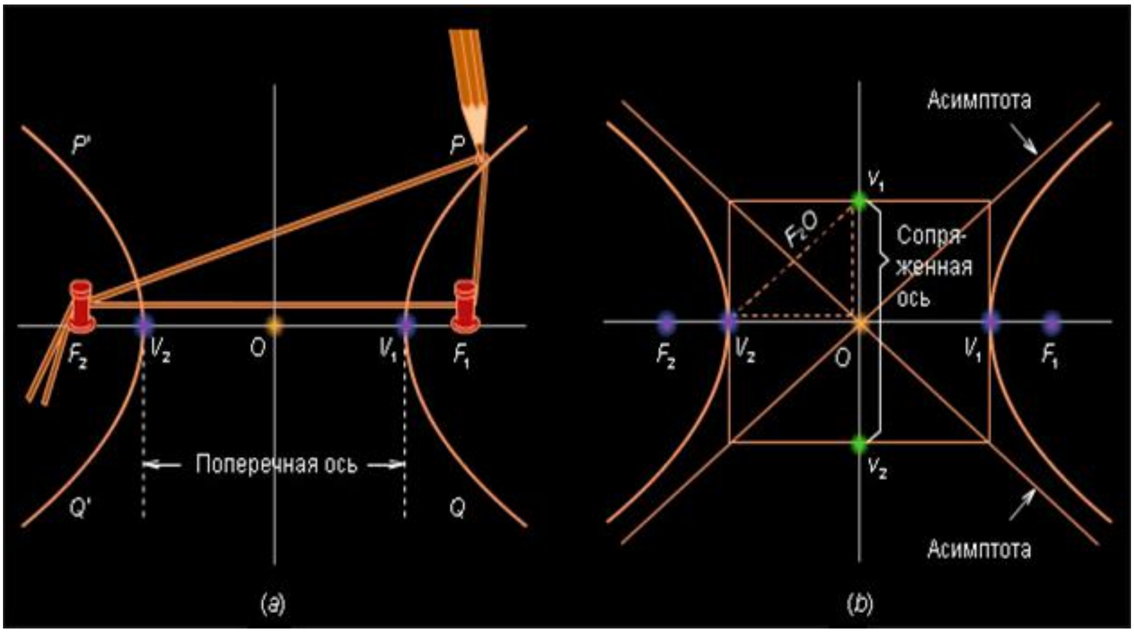
Ellipse. In canonical form with semi-axes a and b, its equation is given as:

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If the ends of a string of a given length are fixed at points F1 and F2 (Fig. 1), then the curve described by the tip of a pencil sliding along the taut string has the shape of an ellipse.



The points F1 and F2 are called the foci of the ellipse, and the segments V1V2 and v1v2 between the points of intersection of the ellipse with the coordinate axes are the major and minor axes. If the points F1 and F2 coincide, then the ellipse becomes a circle.

Hyperbola. When constructing a hyperbola, the point P, the tip of the pencil, is fixed on a string that slides freely over pins installed at points F1 and F2, as shown in Figure 2, a, the distances are chosen so that the segment PF2 exceeds the length of the segment PF1 by a fixed amount less than the distance F1F2. In this case, one end of the string passes under the pin F1, and both ends of the string pass over the pin F2. (The tip of the pencil should not slide along the string, so it must be fixed by making a small loop in the string and threading the tip through it.) We draw one branch of the hyperbola (PV1Q), making sure that the string remains taut all the time, and pulling both ends of the string down behind the point F2, and when the point P is below the segment F1F2, holding the string by both ends and carefully releasing it. We draw the second branch of the hyperbola after first swapping the pins F1 and F2 (Fig. 2).



The branches of the hyperbola $P'V_2Q'$ approach two lines that intersect between the branches. These lines, called the asymptotes of the hyperbola, are constructed as shown in Figure 4, b. The angular coefficients of these lines are $\pm\frac{v_1v_2}{v_1v_2}$. where is the segment of the bisector of the angle between the asymptotes, perpendicular to the segment F2F1; the segment v1v2 is called the conjugate axis of the hyperbola, and the segment V1V2 is its transverse axis.

Thus, the asymptotes are the diagonals of a rectangle with sides passing through the four points v1, v2, V1, V2 parallel to the axes. To construct this rectangle, it is necessary to specify the location of the points v1 and v2. They are equidistant, equal to

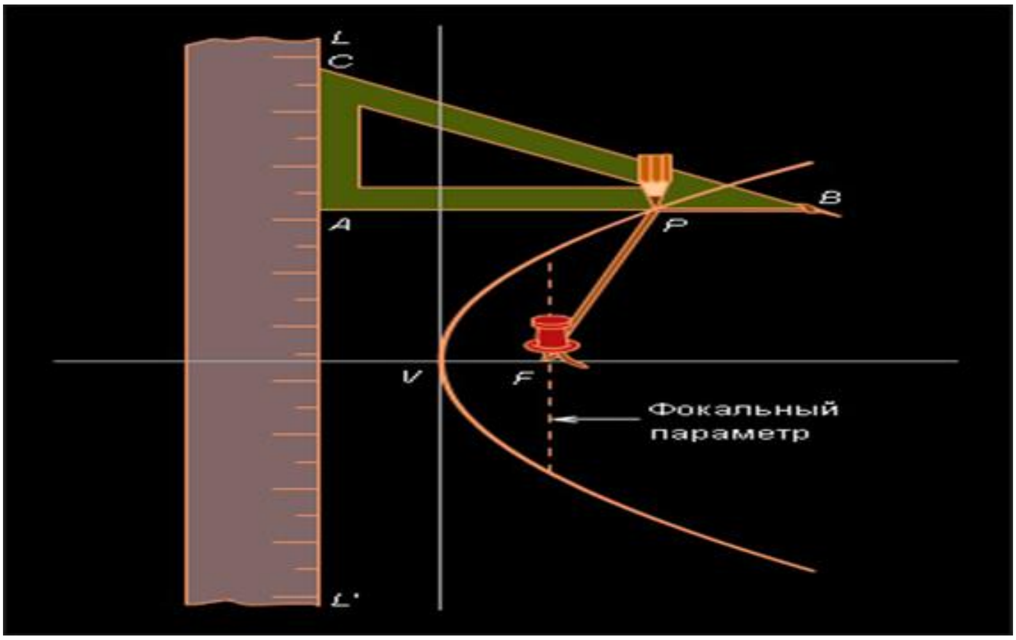
$$Ov_1 = Ov_2 = \sqrt{(F_2O)^2 + (V_2O)^2}$$

from the intersection point of the axes O. This formula assumes the construction of a right triangle with legs Ov1 and V2O and hypotenuse F2O.

If the asymptotes of the hyperbola are mutually perpendicular, then the hyperbola is called equilateral. Two hyperbolas having common asymptotes, but with transposed transverse and conjugate axes, are called mutually conjugate.

Parabola, or, with rotation of coordinates, x^2=*px* the parabola is defined as the locus of points equidistant from the focus and the directrix.

The foci of the ellipse and hyperbola were already known to Apollonius, but the focus of the parabola, apparently, was first established by Pappus (second half of the III century), who defined this curve as the locus of points equidistant from a given point (focus) and a given line, which is called the directrix. The construction of a parabola using a stretched string, based on the definition of Pappus, was proposed by Isidore of Miletus (VI century).



Position the ruler so that its edge coincides with the directrix, and apply the leg AC of the drawing triangle ABC to this edge. Fix one end of a string of length AB at the vertex B of the triangle, and the other end at the focus of the parabola F. Tensioning the string with the tip of the pencil, press the tip at a variable point P to the free leg AB of the drawing triangle. As the triangle moves along the ruler, the point P will describe an arc of the parabola with focus F and directrix, since the total length of the string is AB, the string segment is adjacent to the free leg of the triangle, and therefore the remaining string segment PF must be equal to the remaining part of the leg AB, that is, PA. The intersection point V of the parabola with the axis is called the vertex of the parabola, the line passing through F and V is the axis of the parabola. If a line is drawn through the focus perpendicular to the axis, then the segment of this line cut off by the parabola is called the focal parameter. For an ellipse and hyperbola, the focal parameter is defined similarly.

ANALYTICAL APPROACH

Algebraic classification. In algebraic terms, conic sections can be defined as plane curves whose coordinates in a Cartesian coordinate system satisfy a second-degree equation. In other words, the equation of all conic sections can be written in general form as

(1) $Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$

where not all coefficients A, B, and C are equal to zero. By parallel translation and rotation of axes, equation (1) can be reduced to the form

$ax^2 + by^2 + c = 0$ **or** $px^2 + qy = 0$.

The first equation is obtained from equation (1) when $B^2 > AC$, the second – when $B^2 = AC$. Conic sections whose equations are reduced to the first form are called central. Conic sections defined by equations of the second type with $q > 0$ are called non-central. Within these two categories, there are nine different types of conic sections depending on the signs of the coefficients.

1) If the coefficients a, b, and c have the same sign, then there are no real points whose coordinates would satisfy the equation. Such a conic section is called an imaginary ellipse (or an imaginary circle if a = b).

2) If a and b have one sign, and c has the opposite sign, then the conic section is an ellipse; if a = b – a circle.

3) If a and b have different signs, then the conic section is a hyperbola.

4) If a and b have different signs and c = 0, then the conic section consists of two intersecting lines.

5) If a and b have one sign and c = 0, then there is only one real point on the curve that satisfies the equation, and the conic section is two imaginary intersecting lines. In this case, they also speak of an ellipse contracted to a point or, if a = b, a circle contracted to a point.

6) If either a or b is zero, and the remaining coefficients have different signs, then the conic section consists of two parallel lines.

7) If either a or b is zero, and the remaining coefficients have one sign, then there is not a single real point that satisfies the equation. In this case, they say that the conic section consists of two imaginary parallel lines.

8) If c = 0, and either a or b is also zero, then the conic section consists of two real coincident lines. (The equation does not define any conic section when a = b = 0, since in this case the original equation (1) is not of the second degree.)

9) Equations of the second type define parabolas if p and q are non-zero. If p > 0, and q = 0, we get the curve from item 8. If p = 0, then the equation does not define any conic section, since the original equation (1) is not of the second degree.

Application: Conic sections are often found in nature and technology. For example, the orbits of planets revolving around the Sun are in the shape of ellipses. A circle is a special case of an ellipse, in which the major axis is equal to the minor axis. A parabolic mirror has the property that all incoming rays parallel to its axis converge at one point (the focus). This is used in most reflecting telescopes, where parabolic mirrors are used, as well as in radar antennas and special microphones with parabolic reflectors. A beam of parallel rays emanates from a light source placed at the focus of a parabolic reflector. Therefore, parabolic mirrors are used in powerful searchlights and car headlights. The hyperbola is a graph of many important physical relationships, for example, Boyle's law (relating the pressure and volume of an ideal gas) and Ohm's law, which defines the electric current as a function of resistance at constant voltage.

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