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The Connection Between the Science of Mathematics and the Science of Logic

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Abstract. *The aim of the research is to study the relationship between logic and mathematics in the structure of axiomatized and formalized scientific theories. The object of the research is the explication of this relationship and its explanation. The subject of the research is syntactic and semantic views on the structure of scientific theories; the relationship between logic and mathematics in them has not been studied in detail.*

Key words: *methodology of science, syntactic view, semantic view, mathematics, logic, axiomatic method, formalization, structure of theory, set-theoretic predicate, models.*

In the syntactic view, the structure of the theory is understood as a linguistic construct built from various logical sentences of the theoretical level, correspondence sentences and observation sentences. The structure of the theory does not take into account the diversity of model representations of the theory, which generate many language constructs. The semantic view overcomes this drawback, and in it the structure of the theory is presented as a hierarchy of models: from axioms to theoretical level models, experimental models and data models. The structure of the theory, the relationship between logic and mathematics were studied using comparative analysis, interpretive analysis methods and reconstruction of scientific theories. The methods made it possible to explicate mathematical concepts in the structure of the theory and correlate them with logic and natural language. Comparative analysis showed that in the syntactic view, the connection between logic and mathematics lies in the fact that mathematical concepts of physics are interpreted in the language of first-order predicate logic with equality. The connection between mathematical concepts is provided by the axiomatic method, which serves as a means of formalizing concepts. Mathematics is reduced to logic. In the semantic approach, to identify the connection between mathematics and logic, it was necessary to reconstruct the structure of non-relativistic quantum mechanics. With the help of the Suppes set-theoretic predicate, its axioms were defined, the connection between mathematical structures, postulates of the theory, axioms, and observable quantities was established. Logic and mathematics are related to each other in such a way that metamathematics or linguistics is a part of mathematics. Mathematics includes set theory and model theory, that is, mathematical logic. The connection between mathematical formalisms and phenomena and natural language remains problematic, this shortcoming is also present in the syntactic approach. The novelty lies in the fact that the study contributes to the methodology and logic of science, to the explanation of the connection between logic and mathematics in scientific theory, which was illustrated by various examples from various areas of physics. Logical thinking is a way of thinking in which a person operates with specific concepts, builds reasoning according to certain (logical) laws and draws conclusions based on available data.

Thinking logically means reasoning consistently, drawing conclusions from facts and finding contradictions in your own conclusions or the reasoning of other people. We usually use logical operations - analysis, synthesis, comparison and generalization - without thinking about the names. When reasoning, we go from the general to the particular. Or vice versa, we find the general and the different, combine objects and phenomena according to a common feature. We understand what follows from what, and what information contradicts other data. It's like riding a bicycle: if you learn once, you will always remember and be able to apply the skill in practice. But in order to freely use logical thinking, you need to train it. Mathematics is a science that uses exclusively abstract concepts that have no analogues in the material world. Mathematical reasoning is based on the laws of logic and strictly obeys them. When a child solves a mathematical problem or makes a calculation, he cannot rely on his life experience or intuition, so he needs to use logical thinking.

Most scientists and engineers working in fundamental and applied natural sciences, in technologies, use mainly non-formalized and non-axiomatized mathematical models and theories. By non-formalized models and theories we mean those that are not related to set theory, state space or model theory.

Such are, for example, heuristic models in biology (Watson and Crick's DNA double helix model), sign-symbolic models of structural chemistry, analog models in microphysics (Rutherford's model of the atomic structure), mathematical models and particular theories of physical and chemical processes in the field of kinetics and catalysis, thermodynamics, describing the state of the process at any moment in time using a system of differential equations, simulation models in computer sciences. All these models require insight and logic in constructing a model (analogy, simplification, use of approximations), searching for mathematical analogs of a natural science object, methods for studying the sign patterns of model parameters. Non-formalized and non-axiomatized models and theories can be contradictory, describing the same reality using different representations (Bohr's model of the structure of the atom), their contradictory nature is revealed when their obvious contradictory provisions are revealed, which leads to further detailed analysis of their mathematical and logical structures

For a better understanding of physical reality and the essence of its laws, it is necessary to come to an axiomatized and formalized version of the theory. Axioms allow not only to systematize theoretical knowledge and carry out formal routine verification of evidence, but also to come to new laws, to better understand and establish their connection with physical reality. This is the role of axioms in cognition. Axiomatization allows one to get rid of confusion in the origin of concepts and their description of phenomena, to replace intuitive or semi-empirical concepts of scientific theory with strict mathematical ones, to put an end to thought experiments.

However, the mathematically developed theories of physics in the 19th century, such as thermodynamics and electrodynamics, required a logical justification for their knowledge, which meant formalizing physical theory and explicitly presenting the underlying logical apparatus. It was necessary to improve the logical apparatus and create reliable logical languages.

At the beginning of the 20th century, Hilbert put forward a project to reconstruct the foundations of classical mathematics; his attention was directed to the problem of the consistency of mathematics. To do this, Hilbert proposed to solve it by formalizing all its theories and reducing them consistently to set theory. He wanted to create an axiomatic system that would serve as the foundation of all mathematical theories. Hilbert realized that axiomatization requires well-developed symbolic formal systems with the help of which it would be possible to formalize mathematical theories and proofs. The idea of the program was that it was possible to neglect the semantic meanings of mathematical expressions, replacing them with symbols or strings of algebraic symbols. As a result, mathematical theories had to be replaced by formal systems, and proofs - by sequences of formulas G4, pp- 464 - 4801. Thus, mathematical theories acquired a logical structure, and soon mathematics opened the way to reduction to logic (Russell, Whitehead, Wittgenstein, logical empiricism). Hilbert's project was also aimed at using the formal axiomatic method in mathematical natural science, where it was to become the method of all theoretical research.

Hilbert's program influenced the construction of the axiom system of set theory (Zermelo, von Neumann, Frenkel) and the establishment of its consistency. This required a more in-depth study of the axiom system, which entailed an analysis of the structure of a scientific statement, truth, expressiveness, provability, and the study of mathematical models.

Mathematical logic opened up new perspectives for natural science. Theories of natural sciences appeared as axiomatically formalized systems. The question was to correlate the natural language of the theory, in which empirical facts were described, with their mathematical and logical structures. This question is discussed in syntactic and semantic concepts of science in connection with the problem of reconstructing scientific theories to this day.

Syntactic view: an example from gas molecular-kinetic theory.

The syntactic view (Vienna Circle, logical empiricism) on the subject area of science is that its language, including, for example, physical concepts and natural language, can be expressed with the help of the axiomatic method as a set of propositions in the symbolic logical language for a given subject area J61. The syntactic interpretation of the natural sciences required their formal axiomatic construction according to Hilbert's recipe and the reduction of mathematics to symbolic logic. The theory was a partially interpreted axiomatized system, its language was divided into theoretical language, correspondence language, and observation language. Theoretical propositions are, for example, the laws of thermodynamics, which are composed of theoretical concepts (temperature, pressure, volume), here also belongs the calculus of the adopted axiomatic system, for example, algebra. Correspondence propositions connected propositions of theory with propositions of observation, the latter set the semantics of propositions of theory.

Thus, to explicate the structure of nonrelativistic quantum mechanics, its axioms were defined and interpreted using non-classical logic. Isolation of the stages of theory construction allowed us to better understand the connection between logic and mathematics between mathematical structures and axioms of logic, which consists in the fact that mathematical structures become models of axioms, and the theory itself becomes a family of models. Mathematics includes non-classical logic. Further consideration of the structure of the theory as a hierarchy of models reveals the connection between the axioms, postulates of the theory, its models, models of experiment, and models of data. This entire hierarchical structure is connected by the methodology of the representation theorem.

The focus was on the question of how the basic structures of logic and mathematics participate in the construction of scientific theories and how they relate to the natural language of the theory. For this purpose, a comparative analysis was made between the syntactic and semantic approaches in the philosophy of science. The focus of the study was also the structure of scientific knowledge, which was understood differently by representatives of these approaches. In the syntactic view, the structure of theory was understood as a linguistic construct, and the connection between logic and mathematics was that the theoretical level of cognition included mathematical descriptions of theoretical concepts and laws that were expressed in the language of first-order predicate logic with equality. The connection between mathematical and logical concepts was ensured by the axiomatic method, which acted as a tool for cognition, a means of formalizing mathematical structures. Mathematics was "translated" into the language of symbolic logic. Mathematics was reduced to logic. The theoretical language was linked by means of correspondence sentences to observation sentences, which were analytical judgments S is P, expressed in natural language. Such a structure of the theory was too simple and did not take into account the diversity of model representations of the theory, which generated a multitude of linguistic constructs.

The semantic view tries to overcome this shortcoming. It puts forward two approaches to understanding the structure of the theory: the state space and the theory of models, the latter was developed with the help of the set-theoretic predicate of Suppes, which is the focus of this study to illustrate the connection between logic, mathematics and natural language. For this purpose, a reconstruction of non-relativistic wave mechanics was carried out and it was shown how the construction of a scientific theory occurs and its structure was highlighted.

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