

On the Basics of Education of Mathematical Thinking in the Modern Course of Geometry in a Comprehensive School

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Abstract: *The article focuses on the development of mathematical thinking in students and students of the Pedagogical University, based on the solution of a system of various geometric problems. The ability to create (restore) geometric images used in design is one of the features of functional and mathematical thinking, synthesizing solutions of complex geometric problems that require advanced stereographic imagination from the simplest mental imagery and operations. It also discusses a contemporary methodological issue, namely, the problem of the advanced development of visual thinking. In the current state of mathematical education, this issue has many shades in pedagogical practice one of them is the upbringing of figurative thinking and operational efficiency. Visual thinking is used from kindergarten to mathematics students of pedagogical and technical universities. Another issue posed at the beginning of the last century by the American teacher, psychologist J. Dewey - the possibility of educating thinking and its method - currently has verification in the world pedagogical experience. Works of psychologists, philosophers, lecturers: Pavlova I.P., Bertrand Russell, Bekhterev V.M., Vygotsky L.S., Ilyin E.P., Leontiev A.N., Rubinstein S.L., Luier A.R., Davydova V.V., Korchak Y., Galperina P.Ya., Gessen S.I., Shatalova V.F., Elkoina D.B. and many others elucidated, to one degree or another, the indicated issue, especially interesting in terms of applications to the development or discovery or education of mathematical abilities. This article describes the method of “end- to-end” teaching of mathematicians in a pedagogical university based on issues in geometry.*

Key words: *Education, development, mathematical thinking, verbal-figurative thinking, figurative-verbal thinking, figurative logical thinking, abstract-figurative thinking, abstract- logical thinking, stereographic thinking, concrete thinking.*

INTRODUCTION

It is well known that the upbringing and development of a person begins in the family. The child understands the habits and attitudes of adults from an early age. In addition, its bright colors are attractive. Grasp everything you catch reflexively, as well as react to different sounds and noises. All of these infant-specific behaviors are indicative of a child's healthy reactions [1]. From preschool age, the child is given a poem memorization, classification of colors and shapes, taking into account the age characteristics of the personality. Determine the position of objects in space and the figures in the plane. Related to the words “up”, “down”, “right”, “left”, “forward”, “back”, “behind”, “in front”, “on the side”, “side” perform a variety of actions.

Verbal and logical thinking also develop through these commands and actions. At the preschool age, elementary ideas about mathematical figures and concepts, quantity, size are also created. Such as “more”, “less”, “equal”, “combine” (comparison), “attach”, “break into parts” (cut), “put”, “compare”. As experience shows, from 4-5 years old to 7-8 years old children develop figurative-verbal thinking. During this period, a well-developed child easily perceives the necessary information around him [2, 3].

In elementary school math classes, students begin to form word problems of different content, ways to understand the content of the problem using official language. The educational connection, depending on the teacher’s qualifications, should require the student to use mathematical discourse. Thus, competent mathematical discourse develops. Preconditions are created for the development of verbal-figurative thinking. Over time, from grades 5-6, students in mathematics lessons develop the need to translate texts into images, turning into figurative- logical, and then verbal-logical ideas about tasks [4]. This means that word problems transform figurative pictures and thoughts into an abstract form, represent the meaning (what is required to find) in the problem and decompose its solution into elementary actions and tasks. Starting from the 7th grade, when they begin to study the discipline “Geometry”, they purposefully develop the figurative-logical component of mathematical thinking. The development of figurative-logical thinking is mentioned in [5].

These methods of organizing educational texts contribute to the development of mathematical thinking. Geometric pictures of the world in figurative-verbal and verbal-figurative forms are transformed directly in geometry lessons into a figurative-logical form in which the text of the problem and the text of the student's reasoning are consistent with the rules of the native language and mathematical logic (propaedeutics of the concepts of mathematical logic).

The Main Findings and Results

It should be noted that the age-related features of upbringing and education lead to significant changes in the psychophysiological state of children under 9th grade. In high school and early school years, logical-abstract thinking develops. There are professional aspirations and development programs of students [6, 7, 8].

A few words about the geometric image and the geometric location of points and their methodological usefulness

It is well known that professional thinking, especially engineering thinking, is associated with the “old” type of thinking, i.e., stereographic, volumetric thinking, despite computerization and the development of artificial intelligence. Indeed, before constructing any machine, building or structure, they usually draw their images previously they did it on paper, now they make drawings with the help of “smart” computer programs. But from the conception of the object to its construction, a huge intellectual, constructive work takes place. In order to build a car from the image, the image itself must be sufficiently accurate and meet certain requirements.

In the history of science, the need for a theory of images became clear long ago. Already at the beginning of our era, the Roman historian and architect Vitruvius gives some rules for constructing images. Among the greatest thinkers of mankind who contributed their lempa to the theory of images and the development of geometry as a science were: Leonardo da Vinci (1452-1519), Albrecht Durer (1471-1528). In the history of science, A. Durer’s letter is famous with a request to artists and with a question about their description of the process of depicting objects “I ask everyone who has knowledge or knowledge to make them public ...”. Answers and reflections formed the basis of the books “Guide to Measurement” (about construction with the help of a compass and a ruler), and another book “Four books on proportions.” They formulated the most famous theorems of projective geometry - A. Durer’s theory of perspective “theory is by no means a purely technical discipline, which is forever assigned a purely auxiliary role in painting and architecture, but an important branch of mathematics that has not lost the ability to develop. Indeed, the theory of perspective eventually developed into projective geometry, cited in [9] “Further presentation of the theory of perspective, an outstanding artist of the Renaissance, led him to affine geometry, the

consideration of the affine properties of figures. He much earlier than professional mathematicians approached the theorem, which was formulated by Girard Desargues in his essay on perspective (1636), who also gave strictly scientifically substantiated construction rules in the theory of images. Further, we note that the creation of Gaspard Monge (1745 - 1818) of the method of orthogonal projections, which, without significant changes, is widely used in our time. The military fortifier engineer G. Monge immersed, following R. Descartes and his method, three-dimensional space into two-dimensional using projections.

Drawing plays an important role when studying geometry in a plane. As a rule, the patterns present in a beautifully drawn picture are preserved in high resolution: the middle of the segment is in the middle, the right corner is like a right angle, and so on. Judging by the correctly drawn pictures, the presence of certain patterns can be assumed with sufficient confidence. In solid geometry, the parallel design method is used in the construction of drawings. We have studied this method and its features.

While studying the method of concurrent design, we came across an amazing way to solve a problem. It turns out that any triangle can be designed in such a way that it becomes equilateral, while maintaining the ratio and parallelism of the segments, which will greatly simplify the solution of the problem.

However, the images of spatial figures in an airplane are built according to certain rules, and in a school geometry course they are usually done using the parallel design method, the essence of which is as follows.

Let's give an example:

An arbitrary plane is selected in space, which is called the projection plane or the image plane, and the straight line l intersecting this plane (Fig. 1, a).

Let M' be an arbitrary point of the space. Through this point we draw a line p parallel to l . The point M of intersection of the straight line p with the plane n is called the parallel projection of the point M' onto the plane n in the direction of the straight line l . If M' is a point in the plane n , then M coincides with M' .

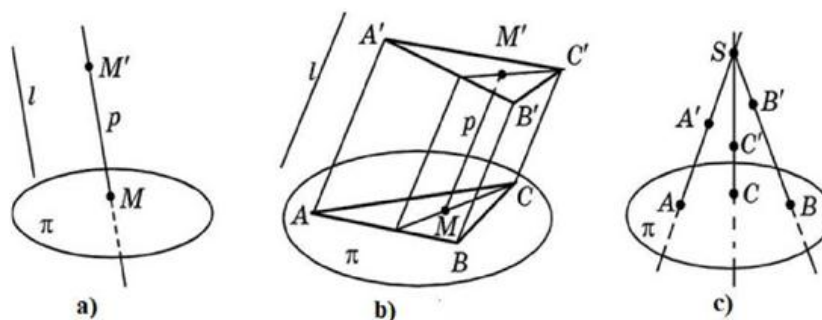


Fig. 1

Line l and all straight spaces parallel to it are called *projecting lines*; they determine the direction of the design. Any plane of space parallel to the projecting line is called the *projecting plane*.

The figure that is designed or depicted is called the *original*. To build a projection of a figure, it is enough to build projections of all points of this figure or projections of points of the figure that define it. In Figure 1, b, triangle ABC is a parallel projection of triangle $A'B'C'$ onto the plane n in the direction of the straight line l .

Comment. Along with parallel design, the *central design* of figures on a plane is also considered. In this case, the projecting straight lines pass through one point — the *projection center*, arbitrarily chosen outside the projection plane (Fig. 1, c).

Parallel and central projection can be observed in real space: the shadow that an object casts on a

sunny day is a parallel projection of this object, since the sun's rays can be considered approximately parallel due to the large distance of the Sun from the Earth. And the image on the cinema screen of the figure filmed on film is the central projection of this figure.

For the theory of constructing images of spatial figures on a plane, an equally important conclusion follows: if the segment $A'C'$ is projected in parallel onto the segment AC and the point B' divides the segment $A'C'$ with respect to $A'B':B'C' = m:n$, then the point B — the projection of the point B' — divides the segment AC in that the same ratio $m:n$, i.e. $AB:BC = A'B':B'C' = m:n$. In particular, the midpoint of the segment $A'C'$ is projected in parallel to the midpoint of the segment AC ($m:n = 1:1$) (Fig. 2).

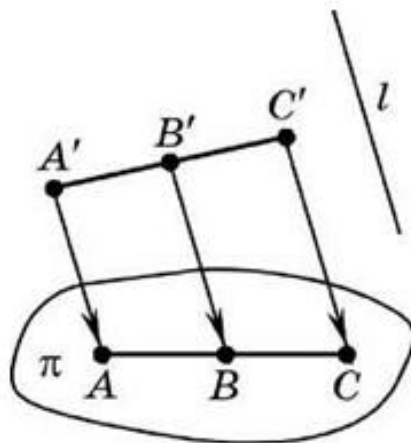


Fig. 2

Let M be an interior point of the segment AB .

Definition: The number λ , equal to the ratio of the lengths of the segments AM and MB , into which the point M divides the segment AB , is called the simple ratio of three points A , B and M lying on one straight line, and is denoted by $(AB; M)$, that is, $(AB; M) = \lambda = AM:MB$

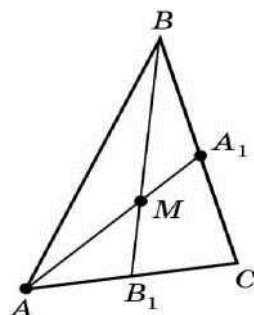


Fig. 3

In this case, points A and B are called basic, and point M is called a dividing point.

The ordering of the points of a simple relation is necessary. For example, if AA_1 is the median of triangle ABC , M is its centroid (the point of intersection of the medians of the triangle), then $(AA_1M) = AM:MA_1 = 2:1$, but $(A_1AM) = A_1M:MA = 1:2$ (Fig.3). Therefore, if $AM \neq MA_1$, then $(AA_1M) \neq (A_1AM)$.

Considering the property of parallel design, we can conclude: **the simple ratio of three points lying on one straight line is preserved in parallel design.** In this case, it is also said that a **simple ratio of three points lying on one straight line** is an *invariant of parallel projection*

The shape properties that are preserved across the parallel design are called the shape's *affine properties*. For example, the properties of lines to be parallel are affine properties of these lines;

invariance of a simple ratio of three points of one straight line is an affine property of such points.

CONCLUSION

In short, the basis of visualizing abstract knowledge is the student's ability to "work" with the simplest things, to construct them, to consider them in motion, in projection.

This skill includes the ability to perform elementary lessons from kindergarten, the ability to cut and apply geometric shapes to each other, to measure in high school, to get acquainted with the concept of measurability of immeasurability "manually", to find out the origin of the multiplicity of measurement methods and a gradual transition to the limiting transition to the idea of rectifiability of arcs and surfaces in the course of geometry at the university, as well as discover the possibilities of design and axonometric representations of geometric bodies. In terms of having the ability to "see" any abstract thing with their own eyes, it will be especially valuable for future biologists, chemists, ecologists, especially if the modern capabilities of computer technology make it easy to obtain the impression about the object, the stereographic thinking of the researcher is the main manager of the reality of the research results.

The methodological effect of such training is that future mathematics teachers will have the opportunity to get acquainted, first of all, with the origin of geometric construction and the theoretical foundations of the projection part of geometry, as the history of geometry has shown necessary to visualize objects.

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