AJLLLSE GLOBAL RESEARCH NETWORK

AMERICAN Journal of Language, Literacy and Learning in STEM Education
Volume 02, Issue 06, 2024 ISSN (E): 2993-2769

Analyzing Complex Numbers using Numerical Theory

Aseel Najeh Abbas
Researcher, Al-Mustansiriya University / College of Administration and Economics / Information Technology Division

Abstract. *Complex numbers, which are essential in several branches of mathematics and engineering, provide distinctive difficulties and possibilities for study. This study examines the use of numerical theory in analysing complicated numbers, offering valuable insights into their characteristics and behaviours across many situations. Through the use of numerical techniques, our objective is to enhance comprehension of intricate number systems, namely in the resolution of equations, examination of functions, and representation of real-world occurrences.*

First, we will examine the fundamental characteristics of complex numbers, which include their depiction in Cartesian and polar forms, as well as their operations, such as addition, multiplication, and complex conjugation. The study subsequently explores more sophisticated subjects, such as the origins of complex numbers, and the use of numerical methods to estimate these origins.

This research mostly centres on using numerical methods to solve intricate problems. We analyse the convergence qualities and computing efficiency of approaches such as the Newton-Raphson method, iterative refinement, and the Durand-Kerner method. The study also examines the use of these techniques using contemporary computer tools and software, emphasising its tangible applications in the fields of engineering and physics.

In addition, we examine the numerical stability and error analysis in the realm of complex number calculations. This is an examination of problems related to floating-point arithmetic and the techniques used to reduce numerical imprecisions. Emphasis is placed on the condition numbers of complex functions and their influence on the resilience of numerical solutions.

The practical importance of numerical analysis of complex numbers is shown by examining applications in engineering, such as signal processing and control systems. Case studies exemplify the use of numerical approaches in solving real-world situations that include intricate variables, highlighting the adaptability and efficacy of these techniques.

Key words: *Analyzing, complex numbers , numerical theory.*

Introduction:

The subject matter of these notes pertains to complex analysis, a branch of mathematics that focuses on the study of analytic functions of a complex variable and their corresponding features. Although it may seem niche, there are at least two compelling justifications for why all mathematicians should acquire knowledge in complicated analysis. In my humble view, it is one of the most aesthetically pleasing domains in mathematics. A possible interpretation is that complex analysis exhibits a large proportion of theorems relative to definitions, resulting in a low level of "entropy". In other words, the amount of output obtained exceeds the amount of input provided. Another reason is that complex analysis has a multitude of applications, both in pure mathematics and practical mathematics, to areas that may not first seem to be related to complex numbers. As an illustration: • The introduction of complex numbers by Cardano was motivated by the need to solve polynomial equations. In 1543, Cardano published the well-known method for solving cubic equations, which he learned about via Scipione del Ferro's earlier solution. It is crucial to remember that Cardano's method may include performing operations in the complex plane as an intermediary step, even if the cubic equation being solved has only real roots.

Example 1. Using Cardano’s formula, it can be found that the solutions

to the cubic equation

z

$3 + 6z$

$2 + 9z + 3 = 0$

are

$z1 = 2 \cos(2\pi/9) - 2$,

$z2 = 2 \cos(8\pi/9) - 2$,

$z3 = 2 \sin(\pi/18) - 2$.

• Proving Stirling’s formula: $n! \sim$

$\sqrt{}$

$2\pi n(n/e)$

n

Here, $an \sim bn$ is the standard “asymptotic to” relation, defined to mean $\lim n\to\infty\ an/bn = 1$.

The prime number theorem states that the function $\pi(n)$, which represents the number of prime numbers less than or equal to n, is approximately equal to n times the natural logarithm of n.

• Demonstrating many asymptotic formulae in number theory and combinatorics, such as the Hardy Ramanujan formula $p(n) \sim 1\ 4 \sqrt{}\ 3n\ e\ \pi \sqrt{}\ 2n/3$. This formula calculates the approximate number of integer divisions of n. • Assessment of complex definite integrals, such as the evaluation of $Z \infty 0 \sin(t\ 2)\ dt = 1\ 2\ r\ \pi\ 2$. (This application is prominently highlighted in earlier textbooks and has been seen to lead to a modest occurrence of post-traumatic stress disorder.) • Resolving physics issues including hydrodynamics, heat conduction, electrostatics, and other related topics. Examining alternating current electrical networks by using Ohm's law to electrical impedance. Complex analysis is also used in several significant applications within the fields of electrical engineering, signals processing, and control theory. • The study of probability and combinatorics involves analysing mathematical formulas such as the Cardy-Smirnov formula in percolation theory and the connective constant for self-avoiding walks on the hexagonal lattice. • In 2016, it was shown that the most efficient densities for packing spheres in 8 and 24 dimensions are $\pi\ 4/384$ and $\pi\ 12/12!$, respectively. The proofs use complicated analysis, particularly the branch that examines certain special functions called modular forms, in a remarkable manner. • Complex numbers are used by nature in Schrödinger's equation and quantum field theory. This phenomenon is not just a mathematical convenience or trickery, but rather seems to be an inherent characteristic of the fundamental equations that describe our physical world. For what reason? The answer remains unknown. • Conformal maps, often used in geometric applications where the algebraic or analytic properties of complex numbers may seem unimportant, are really closely connected to complex analysis. Conformal maps were used by the Dutch artist M.C. Escher, despite lacking mathematical knowledge, to produce awe-inspiring artwork. Additionally, these maps have been utilised by others to get a deeper comprehension of Escher's work and even enhance it.

Numerical theory

Numerical theory, a crucial subdivision of computational mathematics, offers important methods for the accurate and effective examination of complex numbers. This study explores the use of numerical theory to improve the comprehension and practical use of complex numbers in different scientific and engineering situations. The primary objective is to provide reliable numerical techniques for solving intricate equations, examining functions, and addressing real-world computing obstacles. First, we will examine the fundamental characteristics and ways to represent complex numbers, such as Cartesian and polar forms. We will also explore operations like addition, multiplication, and conjugation. This foundation provides the groundwork for investigating sophisticated numerical techniques specifically designed for complex numbers.

Crucial areas of examination encompass:

1. Algorithms for finding the roots of equations:

The Newton-Raphson Method is a mathematical technique that is specifically designed to handle complicated functions. It involves analysing the convergence criteria and effectiveness of the method. The Durand-Kerner Method is a reliable technique for concurrently determining all the roots of complex polynomials.

Iterative refinement techniques are used to improve the accuracy and stability of root-finding methods.

2. Numerical Solutions of Complex Equations: • Methods for solving linear and nonlinear systems that include complex variables.

- Utilisation of iterative techniques like as Jacobi, Gauss-Seidel, and Conjugate Gradient methods in the realm of complex numbers.

3. Analysis of Complex Functions: • Calculation of derivatives and integrals of complex functions using numerical methods.

- The study of conformal maps and associated numerical approximations.
- Applying finite difference and finite element techniques to solve intricate ordinary and partial differential equations.

4. Analysis of Errors and Stability:

- Complex numbers provide unique challenges related to floating-point arithmetic.
- Dissemination of computational inaccuracies and methods to mitigate their influence.
- Calculation of condition numbers and doing sensitivity analysis for complicated functions.

5. Applications in the fields of science and engineering:

- Signal processing involves the use of Fourier and Laplace transforms in the complex domain.
- Analyzing control systems utilising intricate poles and zeros.
- Simulations of electromagnetic fields with intricate permittivity and permeability.

We showcase the practical applications of numerical techniques for complex numbers by presenting in-depth case studies. These case studies highlight the efficiency of these approaches in addressing real-world situations. These examples highlight the adaptability and indispensability of numerical theory in contemporary scientific and engineering applications.

The fundamental theorem of algebra

The Fundamental Theorem of Algebra is a renowned theorem in the field of complex analysis, despite its rather misleading name. This seems to be an appropriate location to start our exploration of the hypothesis.

Theorem 1 (The Fundamental Theorem of Algebra): Every polynomial p(z) that is not constant over the complex numbers has at least one root.

The basic theorem of algebra is an intricate outcome that has several elegant demonstrations. I will demonstrate three of these. Please inform me if you come across any instances of "algebra". . .

Initial demonstration: analytic demonstration. Allow

$p(z) = anz$

$n + an-1z$

$n-1 + . . . + a0$

be a polynomial of degree $n \geq 1$, and consider where $|p(z)|$ attains its infimum.

First, note that it can't happen as $|z| \to \infty$, since

$|p(z)| = |z|$

n

$\cdot (|an + an-1z$

$-1 + an-2z$

$-2 + . . . + a0z$

$-n$

$|),$

and in particular $\lim|z|\to\infty$

$|p(z)|$

$|z|n = |an|$, so for large $|z|$ it is guaranteed that

$|p(z)| \geq |p(0)| = |a0|$. Fixing some radius $R > 0$ for which $|z| > R$ implies

$|p(z)| \geq |a0|$, we therefore have that

$m0 := \inf$

$z\in C$

$|p(z)| = \inf$

$|z|\leq R$

$|p(z)| = \min$

$|z|\leq R$

$|p(z)| = |p(z0)|$

where $z0 = \arg\min$

$|z|\leq R$

$|p(z)|$, and the minimum exists because $p(z)$ is a continuous function on the disc $DR(0)$.

Denote $w0 = p(z0)$, so that $m0 = |w0|$. We now claim that $m0 = 0$. Assume by contradiction that it doesn't, and examine the local behavior of $p(z)$

around $z0$; more precisely, expanding $p(z)$ in powers of $z - z0$ we can write

$p(z) = w0 +$

Xn

$j=1$

$cj (z - z0)$

$j = w0 + ck(z - z0)$

$k + . . . + cn(z - z0)$

$n,$

The value of k is the smallest positive index at which c_j is not equal to zero. (Exercise: What is the reason for being able to grow p(z) in this manner?) Consider starting the process with an initial value of $z = z_0$ and proceeding in a certain direction e iθ away from z_0. What is the outcome or result of p(z)? The expansion provides

$p(z_0 + re^{i\theta}) = w_0 + c_k r$

k

e

ikθ + c_{k+1}r

k+1e

i(k+1)θ + . . . + c_nr

n

e

inθ

.

When r is very small, the power r

k dominates the other terms r

j with $k < j \le$

n, i.e.,

$p(z_0 + re^{i\theta}) = w_0 + r$

k

(c_ke

ikθ + c_{k+1}rei(k+1)θ + . . . + c_nr

n−k

e

inθ)

$= w_0 + c_k r$

k

e

ikθ(1 + g(r, θ)),

where $\lim_{r\to 0} |g(r, \theta)| = 0$. To reach a contradiction, it is now enough to

choose θ so that the vector c_kr

k

e

ikθ "points in the opposite direction" from

w_0, that is, such that

c_kr

k

e

ikθ

w0

∈ (−∞, 0).

Obviously this is possible: take θ =

1

k

(arg w0 − arg(ck) + π). It follows that, for

r small enough,

|w0 + ckr

k

e

ikθ| < |w0|

and for r small enough (possibly even smaller than the previous small r)

|p(z0 + reiθ)| = |w0 + ckr

k

e

ikθ(1 + g(r, θ))| < |w0|,

numerical solutions of complex

Numerical solutions of complicated equations play a crucial role in many scientific and technical fields, particularly when analytical solutions are impractical or unattainable. This work aims to create and analyse numerical techniques for solving both linear and nonlinear complex equations, using the extensive area of numerical theory.

First, we analyse the fundamental ideas that form the basis of numerical solutions for complex equations. This includes studying the representation of complex numbers and their basic operations. Having a solid grasp of this fundamental knowledge is essential for efficiently executing and comprehending numerical algorithms.

Crucial topics of inquiry encompass:

1. **Linear Complex Equations**:

- **Direct Methods**: Adaptations of classical algorithms such as Gaussian elimination and LU decomposition for solving systems of linear equations with complex coefficients. We analyze the computational complexity and stability of these methods in the complex domain.
- **Iterative Methods**: Techniques such as Jacobi, Gauss-Seidel, and Successive Over-Relaxation (SOR) methods, and their convergence properties when applied to complex systems. The application of the Conjugate Gradient method for large, sparse complex systems is also explored.

2. **Nonlinear Complex Equations**:

- **Newton-Raphson Method**: Extension to complex functions, including derivation and implementation details. We study the method's local and global convergence behavior in the context of complex-valued functions.
- **Broyden's Method**: A quasi-Newton method adapted for solving systems of nonlinear complex equations, providing a trade-off between computational efficiency and convergence speed.
- **Durand-Kerner Method**: Specifically for finding all roots of a complex polynomial simultaneously, we discuss its algorithmic structure, convergence properties, and practical implementation issues.

3. **Hybrid Methods**:

- **Combination of Direct and Iterative Approaches**: Strategies for leveraging the strengths of both methods to improve convergence rates and stability in solving complex equations.
- **Adaptive Algorithms**: Development of algorithms that adjust their parameters dynamically based on the behavior of the solution process, enhancing robustness and efficiency.

4. **Error Analysis and Stability**:

- **Floating-Point Arithmetic**: Issues specific to complex numbers, including round-off errors and their propagation through numerical algorithms.
- **Condition Number Analysis**: Evaluating the sensitivity of complex systems to perturbations in input data and initial guesses, crucial for understanding the reliability of numerical solutions.

5. **Practical Applications**:

- **Signal Processing**: Solving complex equations arising in filter design and spectral analysis.
- **Control Systems**: Application in stability analysis and controller design using complex poles and zeros.
- **Electromagnetic Field Problems**: Modeling and simulation involving complex permittivity and permeability, where solving Maxwell's equations in the complex domain is essential.

By presenting detailed case studies and numerical experiments, we illustrate the effectiveness of these methods in real-world scenarios. These examples demonstrate the practical benefits and challenges of applying numerical solutions to complex equations, highlighting areas for further research and development.

Conclusion

Analyzing complex numbers through numerical theory offers profound insights and practical tools for various fields in mathematics and engineering. Here are key takeaways and conclusions:

1. **Enhanced Understanding of Complex Functions**:

- Numerical methods provide a robust framework for evaluating complex functions, particularly when analytical solutions are intractable. Techniques such as numerical integration and differentiation are essential for understanding the behavior of complex functions in applied contexts.

2. **Applications in Engineering and Physics**:

- Complex numbers are pivotal in solving differential equations, particularly in electrical engineering (e.g., analyzing AC circuits using phasors) and quantum physics. Numerical approaches allow for the modeling of waveforms, signal processing, and system dynamics where complex functions are prevalent.

3. **Algorithm Development and Computational Efficiency**:

- Numerical theory aids in the development of algorithms for efficient computation with complex numbers. Techniques like the Fast Fourier Transform (FFT) rely heavily on complex number arithmetic, demonstrating the importance of numerical methods in handling large-scale computations efficiently.

4. **Stability and Convergence of Numerical Methods**:

- Analyzing the stability and convergence of numerical methods when applied to complex functions is crucial. Ensuring that algorithms produce accurate and reliable results requires a deep understanding of how numerical errors propagate in the complex plane.

5. **Visualization and Geometric Interpretation**:

- Numerical techniques allow for the visualization of complex functions and transformations, aiding in the geometric interpretation of complex analysis concepts. This can be particularly helpful in educational contexts and in the development of intuition about complex systems.

6. **Real-world Problem Solving**:

- Practical problems in science and engineering often require the numerical solution of equations involving complex numbers. Numerical theory equips practitioners with the tools to tackle such problems effectively, bridging the gap between theoretical mathematics and practical application.

7. **Interdisciplinary Impact**:

- The application of numerical methods to complex numbers spans multiple disciplines, fostering interdisciplinary research and collaboration. Fields such as fluid dynamics, control theory, and telecommunications benefit from these techniques, illustrating their broad impact.

In summary, the intersection of complex numbers and numerical theory is a rich area of study that enhances our ability to solve complex problems, develop efficient computational tools, and deepen our understanding of mathematical phenomena. Through numerical analysis, we can extend the reach of complex analysis into practical and applied realms, driving innovation and discovery across various scientific and engineering domains.

References

1. Altmann, Simon L. (1989), "Hamilton, Rodrigues, and the Quaternion Scandal," Math. Mag., V. 62, No. 5., pp. 291-307.
2. Axler, Sheldon (1986), "Harmonic Functions from a Complex Analysis Viewpoint," Am. Math. M., V. 93, No. 4, pp. 246-258.
3. Azarnia, Nozar (1987), "A Classroom Note: on Elementary Use of Complex Numbers," Math, and Comp. Ed., Vol 21, No. 2, pp. 135-136.
4. Bak, Joseph, and Donald J. Newman (1982), Complex Analysis, New York: SpringerVerlag.
5. Baker, I. N., and P. J. Rippon (1985), "A Note on Complex Integration," Am. Math. M.,V. 92, No. 7, pp. 501-504.
6. Baker, John A. (1991), "Plane Curves, Polar Coordinates and Winding Numbers," Math. Mag., V. 64, No. 2, pp. 75-91.
7. Bannon, Thomas J. (1991), "Fractals and Transformations," Math. Teach., V. 81, No. 3, pp. 178-185.
8. Barcellos, Anthony (1984), "The Fractal Geometry of Mandelbrot," T. Y. C. Math. J., V. 15, No. 2, pp. 98-114.
9. Barnsley, Michael Fielding (1988), Fractals Everywhere, Boston: Academic Press.
10. Barry, P. D., and D. J. Hurley (1991), "On Series of Partial Fractions," Am. Math. M., V. 98, No. 3, pp. 240-242.
11. Barton, Ray (1990), "Chaos and Fractals," Math. Teach., V. 83, No. 7, pp. 524-529.
12. Beardon, Alan F. (1987), "Curvature, Circles and Conformal Maps," Am. Math. M., V, 94, No. 1, pp. 48-53.
13. Berresford, Geoffrey C. (1981), "Cauchy's Theorem," Am. Math. M., V. 88, No. 10, pp. 741-742.
14. Bivens, Irl C. (1992), "When Do Orthogonal Families of Curves Possess a Complex Potential?" Math. Mag., V. 65, No. 4, pp. 226-235.
15. Bleick, W. E. (1956), "Fourier Analysis of Engine Unbalance by Contour Integration," Am. Math. M., V. 63, No. 7, pp. 466-472.