

## IN HIGHER MATHEMATICS, THE EXTREMUM OF A MULTIVARIABLE FUNCTION

*Madina Polatovna Sharipova*

*Teacher of the "General Technical Sciences" department of Asia International University*

**Abstract:** In higher mathematics, the extremum of a multivariable function is a crucial concept with wide-ranging applications in fields such as optimization and physics. This article aims to provide an overview of the extremum of a multivariable function, exploring the role of critical points, the Second Derivative Test, and Lagrange multipliers in determining the nature of extrema. The article also delves into the significance of extremum in real-world problem-solving, emphasizing the practical implications of this mathematical concept.

**Keywords:** Extremum, multivariable function, critical points, Second Derivative Test, Lagrange multipliers, optimization, mathematics, real-world applications.

In the realm of higher mathematics, the concept of an extremum is fundamental to the study of multivariable functions. Multivariable functions are functions with more than one input variable, and finding their extrema is essential for addressing a wide array of problems in diverse domains, ranging from engineering and economics to physics and computer science. Extrema, which include both maximum and minimum values, play a crucial role in optimization, where the objective is to either minimize or maximize a certain quantity subject to given constraints. In this article, we will explore the theory and applications of extrema in multivariable functions, shedding light on the methodologies used to identify and characterize these critical points. In the realm of higher mathematics, the concept of extremum plays a crucial role in understanding the behavior and properties of multivariable functions. An extremum refers to the maximum or minimum value of a function within a certain domain, and it is a fundamental concept in optimization, critical points, and the study of local and global properties of functions.

One of the key steps in identifying extrema is to find the critical points of a multivariable function. Critical points occur where the gradient of the function is either zero or undefined. By setting the partial derivatives of the function equal to zero, the critical points can be determined, providing the initial insights into potential extrema. For a multivariable function, which takes multiple input variables and produces a single output, the process of finding extremum involves the analysis of critical points and the use of mathematical tools such as partial derivatives, gradients, and Hessian matrices.

Second Derivative Test:

The Second Derivative Test is a critical tool for determining the nature of the critical points found. By analyzing the determinant of the Hessian matrix – a matrix of second partial derivatives – at each critical point, one can classify these points as local maxima, local minima, or saddle points. This test enables mathematicians and scientists to ascertain the precise nature of the extrema and make informed decisions in real-world scenarios.

## Lagrange Multipliers:

In the context of constrained optimization, Lagrange multipliers offer a powerful technique for identifying extrema subject to certain constraints. By introducing Lagrange multipliers, which act as a set of additional variables, the extrema of a multivariable function can be found under specified constraints. This method is widely used in fields such as economics, physics, and engineering to optimize functions within given limitations.

The first step in identifying the extremum of a multivariable function is to locate its critical points. These are the points where the partial derivatives of the function with respect to each input variable are zero or undefined. Mathematically, a critical point of a function  $f(x, y)$  occurs at the point  $(x_0, y_0)$  if the partial derivatives  $\partial f / \partial x$  and  $\partial f / \partial y$  are both zero at that point, or if they are undefined.

Once the critical points are identified, the second derivative test can be used to determine whether these points correspond to a local maximum, minimum, or saddle point. The second derivative test involves the calculation of the Hessian matrix of the function, which is a matrix of second-order partial derivatives. By examining the eigenvalues of the Hessian matrix at the critical points, it is possible to classify the nature of the extremum at each point.

If the Hessian matrix has all positive eigenvalues at a critical point, then the point corresponds to a local minimum of the function. Conversely, if the Hessian matrix has all negative eigenvalues, the point corresponds to a local maximum. If the eigenvalues are of mixed signs, the point corresponds to a saddle point, where the function neither reaches a maximum nor a minimum.

In addition to local extremum, one can also analyze the global extremum of a multivariable function over a specific domain. This involves examining the behavior of the function at the boundaries of the domain as well as at critical points within the domain. Through the use of Lagrange multipliers and other optimization techniques, it is possible to find the global maximum or minimum of a function over a given region.

Understanding the extremum of a multivariable function is essential in various fields such as economics, physics, engineering, and computer science, where optimization and maximizing or minimizing certain quantities are common tasks. By grasping the intricacies of extremum in higher mathematics, researchers and practitioners can make more informed decisions and predictions based on the behavior of complex multivariable functions.

In the realm of higher mathematics, the study of extrema in multivariable functions provides invaluable insights into the behavior and optimization of complex systems. The concepts of critical points, the Second Derivative Test, and Lagrange multipliers are indispensable tools for addressing real-world problems and enhancing our understanding of mathematical phenomena. By gaining a comprehensive understanding of these concepts, mathematicians and scientists can leverage the power of multivariable functions to tackle various challenges and contribute to the advancement of various disciplines.

## References

1. Sharipova, M. P. L. (2023). CAPUTA MA'NOSIDA KASR TARTIBLI HOSILALAR VA UNI HISOBBLASH USULLARI. Educational Research in Universal Sciences, 2(9), 360-365.
2. Sharipova, M. P. (2023). MAXSUS SOHALARDА KARLEMAN MATRITSASI. Educational Research in Universal Sciences, 2(10), 137-141.

3. Madina Polatovna Sharipova. (2023). APPROXIMATION OF FUNCTIONS WITH COEFFICIENTS. American Journal of Public Diplomacy and International Studies (2993-2157), 1(9), 135–138.
4. Madina Polatovna Sharipova. (2023). Applications of the double integral to mechanical problems. International journal of sciearchers,2(2), 101-103.
5. Sharipova, M. P. L. (2023). FINDING THE MAXIMUM AND MINIMUM VALUE OF A FUNCTION ON A SEGMENT. American Journal of Public Diplomacy and International Studies (2993-2157), 1(9), 245-248.
6. Sharipova, M. P. (2023). FUNKSIYALARNI KOEFFITSIENTLAR ORQALI FUNKSIYALARNI YAKINLASHTIRISH HAQIDA MA'LUMOTLAR. GOLDEN BRAIN, 1(34), 102–110.
7. qizi Latipova, S. S. (2023). KASR TARTIBLI HOSILA TUSHUNCHASI. SCHOLAR, 1(31), 263-269.
8. qizi Latipova, S. S. (2023). RIMAN-LUIVILL KASR TARTIBLI INTEGRALI VA HOSILASIGA OID AYRIM MASALALARING ISHLANISHI. Educational Research in Universal Sciences, 2(12), 216-220.
9. qizi Latipova, S. S. (2023). MITTAG-LIFFLER FUNKSIYASI VA UNI HISOBBLASH USULLARI. Educational Research in Universal Sciences, 2(9), 238-244.
10. Shahnoza, L. (2023, March). KASR TARTIBLI TENGLAMALARDA MANBA VA BOSHLANG'ICH FUNKSIYANI ANIQLASH BO'YICHA TESKARI MASALALAR. In "Conference on Universal Science Research 2023" (Vol. 1, No. 3, pp. 8-10).
11. Latipova, S. S. qizi . (2023). BETA FUNKSIYA XOSSALARI VA BU FUNKSIYA YORDAMIDA TURLI MASALALARNI YECHISH. GOLDEN BRAIN, 1(34), 66–76.
12. Jurakulov, SZ (2023). NUCLEAR ENERGY. Educational Research in Universal Sciences , 2 (10), 514-518.
13. Oghly, JSZ (2023). PHYSICO-CHEMICAL PROPERTIES OF POLYMER COMPOSITES. American Journal of Applied Science and Technology , 3 (10), 25-33.
14. Oghly, JSZ (2023). THE RELATIONSHIP OF PHYSICS AND ART IN ARISTOTLE'S SYSTEM. International Journal of Pedagogics , 3 (11), 67-73.
15. Oghly, JSZ (2023). BASIC PHILOSOPHICAL AND METHODOLOGICAL IDEAS IN THE EVOLUTION OF PHYSICAL SCIENCES. Gospodarka i Innowacje. , 41 , 233-241.
16. ugli Jurakulov, SZ (2023). STRATEGIES FOR SUCCESSFUL PHYSICS EDUCATION. Educational Research in Universal Sciences , 2 (14), 46-48.
17. Oghly, JSZ (2023). A Japanese approach to in-service training and professional development of science and physics teachers in Japan. American Journal of Public Diplomacy and International Studies (2993-2157) , 1 (9), 167-173.
18. Oghly, JSZ (2023). STRATEGIES FOR SUCCESSFUL LEARNING IN PHYSICS. American Journal of Public Diplomacy and International Studies (2993-2157) , 1 (9), 312-318.
19. Jurakulov, SZO, & Turdiboyev, H. (2023). RELATIONSHIPS OF PHYSICS WITH ART IN THE FIELD OF EDUCATION. GOLDEN BRAIN, 1(33), 144–147.
20. Jurakulov, SZO, & Turdiboyev, K. (2023). STUDYING PHYSICS USING A COMPUTER. GOLDEN BRAIN, 1(33), 148–151.
21. Jurakulov, SZO, & Nurboyev, O. (2023). LEVEL AND POSITION IN THE EDUCATIONAL FIELD OF PHYSICS. GOLDEN BRAIN, 1(33), 157–161.
22. Jurakulov, SZO, & Nurboyev, O. (2023). THE MAIN SIGNIFICANCE OF THE DEPARTMENTS OF PHYSICS IN THE DEVELOPMENT. GOLDEN BRAIN, 1(33), 162–167.

23. Jurakulov, SZO, & Nurboyev, O. (2023). RELATIONSHIPS BETWEEN THE DIRECTIONS OF FINANCE AND PHYSICAL SCIENCE. GOLDEN BRAIN, 1(33), 168–172.

24. Jurakulov, SZO, & Hamidov, E. (2023). PROPERTIES AND CHARACTERISTICS OF NUCLEAR ENERGY. GOLDEN BRAIN, 1(33), 182–186.

25. Jurakulov, SZO, & Turdiboyev, H. (2023). ADVANCED STRATEGIES FOR LEARNING PHYSICS. GOLDEN BRAIN, 1(33), 152–156.

26. Oghly, JSZ (2023). New Computer-Assisted Approaches to Teaching Physics. American Journal of Public Diplomacy and International Studies (2993-2157) , 1 (10), 173-177.

27. Oghly, JSZ (2023). A Current Perspective on the Relationship between Economics and Physics. American Journal of Public Diplomacy and International Studies (2993-2157) , 1 (10), 154-159.

28. Axmedova, Z. I. (2023). LMS TIZIMIDA INTERAKTIV ELEMENTLARNI YARATISH TEXNOLOGIYASI. Educational Research in Universal Sciences, 2(11), 368-372.

29. Ikromovna, A. Z. (2023). USING THE USEFUL ASPECTS OF THE MOODLE SYSTEM AND ITS POSSIBILITIES. American Journal of Public Diplomacy and International Studies (2993-2157), 1(9), 201-205.

30. Axmedova, Z. (2023). MOODLE TIZIMI VA UNING IMKONIYATLARI. Development and innovations in science, 2(11), 29-35.

31. Zulxumor, A. (2022). IMPLEMENTATION OF INTERACTIVE COURSES IN THE EDUCATIONAL PROCESS. ILMIY TADQIQOT VA INNOVATSIYA, 1(6), 128-132.

32. Муродов, О. Т. (2023). РАЗРАБОТКА АВТОМАТИЗИРОВАННОЙ СИСТЕМЫ УПРАВЛЕНИЯ ТЕМПЕРАТУРЫ И ВЛАЖНОСТИ В ПРОИЗВОДСТВЕННЫХ КОМНАТ. GOLDEN BRAIN, 1(26), 91-95.

33. Murodov, O. T. R. (2023). ZAMONAVIY TA'LIMDA AXBOROT TEXNOLOGIYALARI VA ULARNI QO'LLASH USUL VA VOSITALARI. Educational Research in Universal Sciences, 2(10), 481-486.

34. Murodov, O. T. (2023). INFORMATIKA FANINI O'QITISHDA YANGI INNOVATSION USULLARDAN FOYDALANISH METODIKASI. GOLDEN BRAIN, 1(34), 130–139.

35. Sharopova, M. M. qizi . (2023). JAVA TILI YORDAMIDA OB'EKTGA YUNALTIRILGAN DASTURLASH ASOSLARI BILAN TANISHISH. GOLDEN BRAIN, 1(34), 111–119.

36. qizi Sharopova, M. M. (2023). RSA VA EL-GAMAL OCHIQ KALITLI SHIFRLASH ALGORITMI ASOSIDA ELEKTRON RAQMLI IMZOLARI. RSA OCHIQ KALITLI SHIFRLASH ALGORITMI ASOSIDAGI ELEKTRON RAQAMLI IMZO. Educational Research in Universal Sciences, 2(10), 316-319.

37. Jalolov, T. S. (2023). Solving Complex Problems in Python. American Journal of Language, Literacy and Learning in STEM Education (2993-2769), 1(9), 481-484.

38. Jalolov, T. S. (2023). PEDAGOGICAL-PSYCHOLOGICAL FOUNDATIONS OF DATA PROCESSING USING THE SPSS PROGRAM. INNOVATIVE DEVELOPMENTS AND RESEARCH IN EDUCATION, 2(23), 220-223.

39. Tursunbek Sadriddinovich Jalolov. (2023). ARTIFICIAL INTELLIGENCE PYTHON (PYTORCH). Oriental Journal of Academic and Multidisciplinary Research , 1(3), 123-126.

40. Jalolov, T. S. (2023). ADVANTAGES OF DJANGO FEMWORKER. International Multidisciplinary Journal for Research & Development, 10(12).

41. Jalolov, T. S. (2023). ARTIFICIAL INTELLIGENCE PYTHON (PYTORCH). Oriental Journal of Academic and Multidisciplinary Research, 1(3), 123-126.

42. Jalolov, T. S. (2023). SPSS YOKI IJTIMOIY FANLAR UCHUN STATISTIK PAKET BILAN PSIXOLOGIK MA'LUMOTLARNI QAYTA ISHLASH. *Journal of Universal Science Research*, 1(12), 207–215.

43. Tursunbek Sadriddinovich Jalolov. (2023). THE MECHANISMS OF USING MATHEMATICAL STATISTICAL ANALYSIS METHODS IN PSYCHOLOGY. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 138–144.

44. Tursunbek Sadriddinovich Jalolov. (2023). PROGRAMMING LANGUAGES, THEIR TYPES AND BASICS. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 145–152.

45. Tursunbek Sadriddinovich Jalolov. (2023). PYTHON TILINING AFZALLIKLARI VA KAMCHILIKLARI. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 153–159.

46. Tursunbek Sadriddinovich Jalolov. (2023). PYTHON DASTUR TILIDADA WEB-ILOVALAR ISHLAB CHIQISH. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 160–166.

47. Tursunbek Sadriddinovich Jalolov. (2023). SUN'iy INTELLEKTDA PYTHONNING (PYTORCH) KUTUBXONASIDAN FOYDALANISH. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 167–171.

48. Tursunbek Sadriddinovich Jalolov. (2023). WORKING WITH MATHEMATICAL FUNCTIONS IN PYTHON. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 172–177.

49. Tursunbek Sadriddinovich Jalolov. (2023). PARALLEL PROGRAMMING IN PYTHON. *TECHNICAL SCIENCE RESEARCH IN UZBEKISTAN*, 1(5), 178–183.