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Reserves Estimation Using Production Rate Decline Trend Analyses

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Abstract. A constant production rate decline was simulated for reserves estimation, using flow regimes in oil and gas reservoirs. The problem of estimating oil reserves in place or cumulative recovery factor in a dynamic state calls for research. The problem has been so because no researcher has estimated accurate decline constant which can predict an accurate recovery factor or estimate accurate reserves initially in place. Flow regimes in reservoirs were established as a field of study using equations of fluid flow in porous media. The fluid flow equations were used to establish the decline trend, and the estimated production rate decline constant was also used to model the decline. The decline rate constants were subsequently used to build models to predict yearly production data for any given period. These were used to generate models for projecting oil and gas cumulative production and reserves in place. The results obtained were validated using actual yearly production field data from the original data source. The evaluation models were subsequently built using the predicted yearly data. The models were then used to estimate cumulative reserves recovery in any given period and reserves initially in place. The values obtained compared favourably with the respective tank values. The percentage accuracy for gas fields ranged from 99.86% and above, while the percentage accuracy for oil ranged from 98.64% to 99.98%. The models reduce the complexity and time used in reservoir simulation. They are very flexible and can be applied with high accuracy from the reservoir decline stage to abandonment. They are equally used for the establishment of production and for making economic decisions.

Keywords: Production rate, decline constant, reserves estimation, cumulative recovery factor, reserve in place, estimation mode

Introduction

1 The conventional methods for estimating decline constant are decline curve analyses or graphical procedures obtained from existing field production data. Conventionally, decline curve analyses are only possible when the production data or history is available, so that the trend can be defined or delineated. There are no fundamental theoretical trends for decline curve analyses, but the exercise is based on production data history; for this, the principal challenge is to minimize reserves estimation errors, simplify the complex tools used in conventional methods and reduce time consumption. In this research, field production data are used to project yearly production data (called regional or field data) [1]. There are three principal types of decline rate as postulated by the early researchers, namely exponential or constant decline rate, harmonic decline rate and hyperbolic decline rate. The classification was based on constant or variable changes in the factors that influence the fluid flow in a porous medium. Factors that influence the fluid flow in a porous medium are Constant well back pressure effects, Active water-drive influences, Boundary conditions in a porous zone, Historical observation of the data, Single or multiple-phase fluid flow, and combined oil and gas flow as a stream [2]. Oil recovery varies from year to year against the prediction. Horizontal wells are better option for developing reservoirs with oil rim as to conventional wells. Oil recovery is strongly dependent on the oil rim thickness, relative gas-cap size (in-factor), permeability, viscosity and aquifer strength. Hydrocarbon production decline rate as a factional change in the flow rate (q) with respect to time (t). Hyperbolic and Harmonic Decline Rate coefficient of fraction decline when "$0< n <1.0$" [3]. Decline coefficient of the decline rate for harmonic decline is unity ($n = 1$). Equally produced tabulated values needed for plotting hyperbolic type curves using the values of "$0.1 <n <0.9$" with 0.1 incremental values ($n + 0.1+\ldots\ldots n$) [4]. The value of rate decline range is "($0.1<b <0.9$" and about 40% of the leases have the value of rate decline constant greater than 0.5, "($b >0.5$")). In commingled layered reservoirs the values of ‘‘b’’ lie between 0.5 and 1.0. In such a case decline analysis should be initialized from the start of the decline rate [5]. Under certain production and scenarios, initially the rate does not decline. Layered reservoirs can cause the value of ‘‘b’’ to go above unity ("$b >1.0$") Fractional Hydrocarbons Decline Rate, hyperbolic decline rate decrease in oil or gas production per unit time as a fraction of the production rate is proportional to a fraction power called ‘’n’’ ranged: 0.1 "$<n <0$" [6]. Elliptical flow to govern flow regime in a low permeability gas reservoir with elliptical outer binding an elliptical inner boundary surrounded by an elliptic aquifer is an elliptical outer boundary will be a single-layer system that is isotropic, horizontal and uniform thickness decline constant "$b >1.0$" . An appreciably water influx in a gas reservoir acts as pressure maintenance naturally delaying the decline initiation. The benefit is that much of the hydrocarbons are produced [7]. The disadvantage is that such a reservoir is difficult to model, due to less knowledge of the aquifer behavior and life span. Modified Hyperbolic Decline that at some point in time the hyperbolic decline is converted into an exponential decline. Extrapolated hyperbolic decline over long periods of time, frequently results in unrealistically high pressure. To avoid this problem, assumed that for a particular example, the decline rate (D) starts at 30% of flow and declines in a hyperbolic manner [8]. When it reaches a specified value say 10% of the hyperbolic decline it converted to an exponential decline. The error here is that exponential decline rate of 10% would be considered in the forecast. Relative decline rate of about 40% of leases have "$b>0.5$" and commingled layered reservoirs fall between "$0.5<b<1.0$" . Most of attempts on estimated reserves initially in place and the accuracy in DFCA has not been properly delineated. The gap I intent to fill is inaccurate decline constant ‘b’ estimation and improve reserves ("N_p" and N) estimation accuracy from 60/or 67% to about 90 or 99% [9].

Methodology

a. Materials

Fields past production data (called regional data) covering Exploration (wildcat) wells, Appraisal (out-step) wells and Production (exploration development) wells were collated. The first set of data was specifically from the exploration, appraisal and production wells, because those wells could define early-stage to the actual production data records, while the second set of data was from the tanks-farms yearly production records (surface facilities) of the same Niger Delta formation oil wells. These were used mainly for validating the input data. The third set of data were projected rate values (called generic or projected data) were also generated. The main data were early-to-abandonment-stage rates.

Evaluation Models – I: [Governing Models]

Initial rates to abandonment were plotted against time and generated governing evaluation curves. The curves were used to estimate the decline constant, ‘‘b’’ were used and predicted yearly rates, the yearly rates were used to build evaluation models, and the models were then used to estimate the reserves [N_p & N].

Evaluation Models – II:

(i) Initial rates to given periods of production were analyzed for decline constant, ‘‘b’’, the decline constant, ‘‘b’’ was used and predicted yearly rates, the yearly rates were mow used and generated evaluation curves in a given period of production & extrapolated the curves to abandonment, the extrapolated curves were used and built evaluation models and the models were then used and estimated the reserves [N_p & N].

(ii) Short-period production rates were equally analysed for decline trends and constant, ‘‘b’’, the declined constant, ‘‘b’’ was used and predicted yearly rates to abandonment (called generic data), the generic data were used and generated evaluation curves, the curves were used and built evaluation models and the models were then used and estimated the reserves [N_p & N]. Figure 1 below shows a flowchart of the data collation and figure 2 shows the flowchart for quality evaluations and applications.

(iii)

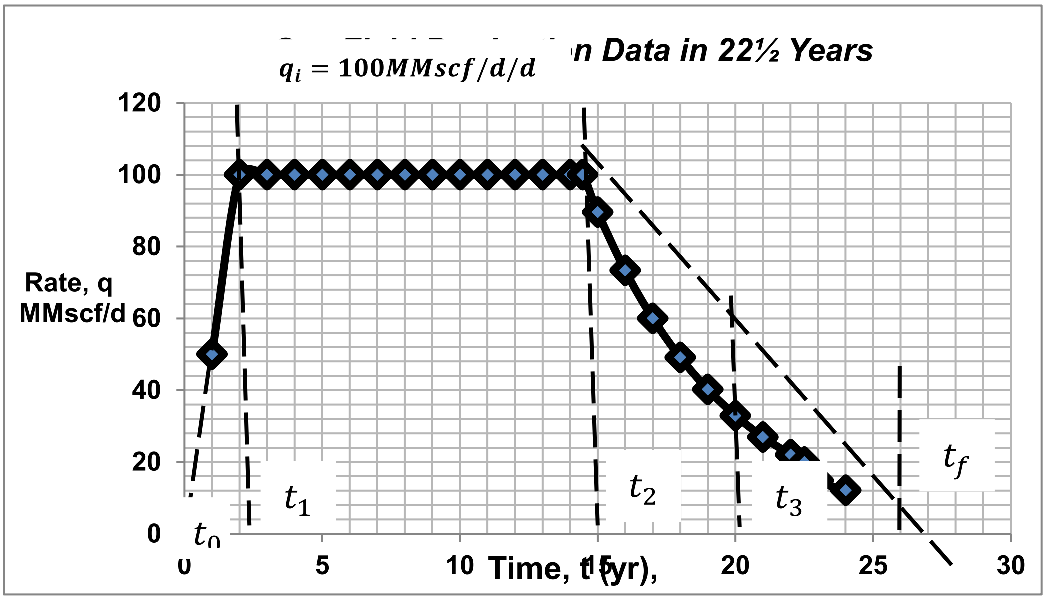
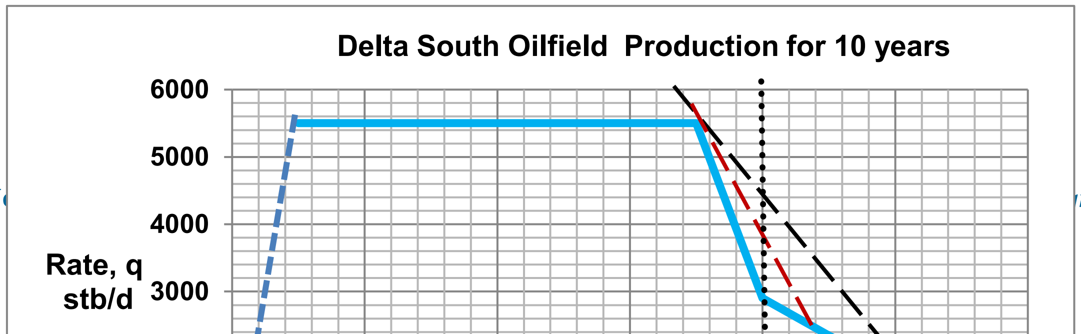


Fig 1 Projectile Decline Curve, using Table 1



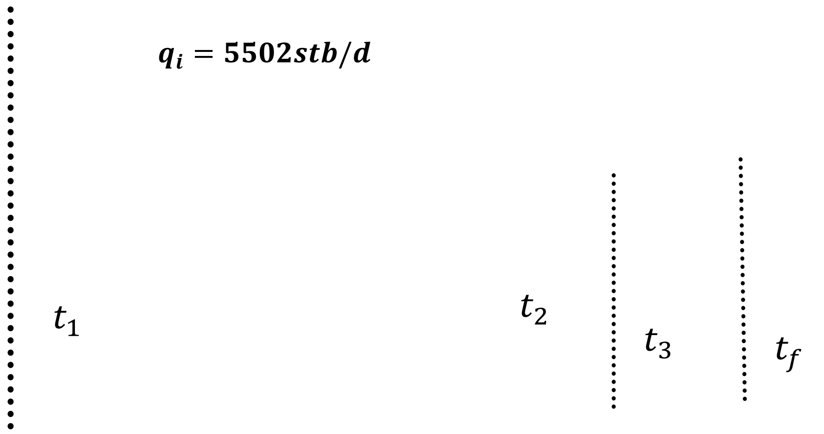


Fig 2 Projectile Decline Curve, using Table 2

Postulation of the Projectile Oil Flow Models

In this section, the principal method for postulating the evaluation models was the projectile-dominated flow of the reserves. The projectile flow was found common in the depletion of saturated hydrocarbons reservoirs from the initial stage to an abandonment stage. Hydrocarbon reserve recovery values in Table 1 were used in plotting the curves, which were used to study the complete reserve recovery from the initial stage through the transient stage, steady stage, the decline stage to economic rate called abandonment rate (Figure 1) and yearly oil recovery data in Table 2 were used to study complete oil recovery (Figure 2). The resulting curves in projectile shapes were used to develop the models for studying the decline trends and projected to both given recovery periods for the cumulative reserves (N_p) and zero decline for estimating the reserves initially in place (N). An oilfield must contain a reserve initially in place (N), which reduces per unit time due to production operations. The flow rate (q) of oil stream production continues to change from time-t_o to time-t_1 and from time-t_1 to time-t_2 and from time-t_2 to time-t_3, (Figure 3), then extrapolated to time-t_f, the initial reserves values. The hydrocarbons production (N_p) per unit time declined from the initial value to minimum $\left(\frac{dq}{dt} = -bq^n\right)$. The constant of proportionality is -b. The quantity of the reserves remaining in the reservoir is N_f. Construction: Pt-B was joined to pt-E, giving the trapezium ABEO, and pt-B to pt-D, giving the trapezium ABDO, respectively. Eqn1 is the general material balance equation (MBE).

Evaluation Model – 1: The Projectile Oil Flow

Using Figure 1 or 2, actual reserves produced in a given period and initially in place are expanded as:

$$\begin{bmatrix}\text{Actual Reserves}\\ \text{Produced in a}\\ \text{Given Time}\end{bmatrix} = \begin{bmatrix}\text{Actual Reserves}\\ \text{Initially in Place}\end{bmatrix} - \begin{bmatrix}\text{Actual Reserves}\\ \text{remaining in Place}\end{bmatrix}$$

$$N_p = N - N_f \qquad 1$$

[Oil Produced]= [Area of the Trapezium ABEO]

[Oil Produced]=$\frac{1}{2}$[Sum of the Parallel Sides]*[Height]

$$N_p = \frac{q_i}{2}\left[(t_2-t_1)+(t_3-t_0)\right] \quad \text{Or} \quad [A_1 + A_2 \quad A_3] \qquad 2$$

$A_1 = \frac{q_i}{2}[t_1-t_0]$= Area of ΔAHO,

$A_2 = q_i[t_2- t_1]$= Area of the rectangle ABFH and

$A_3 = \frac{q_i}{2}[t_3-t_2]$= Area of ΔBEF.

Substituting A_1, A_2 and A_3, gives Eqn3. If you so desired, using equation of the curve part ($A_3 = N_{p3} = \frac{q_i}{b}\left(1-e^{-bt}\right)$) in Figure 3, gives the Oil recovered in Decline rate Stage. This model derived from the first principle below.

$$N_p = \frac{q_i}{2}\left[(t_2- t_1)+ (t_3-t_o)\right] \quad \text{Oil Production} \qquad 3$$

Similarly: $G_p = \frac{q_i}{2}\left[(t_2- t_1)+ (t_3-t_o)\right]$ Gas Production

Yearly Hydrocarbons Production Projection

The general equation for natural production of an oilfield reserves is the product of the rate-constant and the actual rate raised to power-n. This is given mathematically by Eqn4:

$$\begin{bmatrix}\text{Actual Change}\\ \text{in Production}\\ \text{Rate with Time}\end{bmatrix} = \begin{bmatrix}\text{A Decline Rate}\\ \text{Constant}\end{bmatrix}\begin{bmatrix}\text{Actual Rate in}\\ \text{n-order}\end{bmatrix}$$

$$\frac{dq}{dt} = -bq^n \qquad 4$$

Integrating Eqn4, gives the governing equation, Eqn6. The governing equation, Eqn6 is used to postulate actual yearly oil production rate (q) by removing the log and rearranging giving Eqn7. To estimate the rate-constant (b), the governing equation, Eqn6 is rearranged to obtain Eqn7. Three main flow orders of decline rates were considered.

When n=1: 1^{st} Order Decline Rate Parabolic Flow

$$\int_{q_i}^{q}\frac{dq}{q} = -b\int_{o}^{t}dt \qquad 5$$

$$\ln q - \ln q_i = -bt \qquad 6$$

$$q = q_i\, e^{-bt} \quad \text{and} \quad b = \frac{\ln(q_i/q)}{t - t_i} \qquad 7$$

Cumulative reserves Production Models

The general equation for natural production of an oilfield reserves is the product of the hydrocarbons flow rate and the actual time elapsed. This is given mathematically in Eqn8. In Eqn7, $q = q_i e^{-bt}$, so substituting this q in Eqn8 gives Eqn9. Solving Eqn9 gives Eqn10, the governing evaluation models postulated for actual oil cumulative production.

$$\begin{bmatrix}\text{Actual Cumulative}\\ \text{Hydrocarbons}\\ \text{Produced With Time}\end{bmatrix} = \begin{bmatrix}\text{Actual Production}\\ \text{Rate Per Unit Time}\end{bmatrix}\begin{bmatrix}\text{Actual Time}\\ \text{That Elapsed}\end{bmatrix}$$

$$N_p = qdt \qquad 8$$

$$N_p = \int_{o}^{t} q_i e^{-bt}dt \qquad 9$$

$$N_p = \frac{q_i}{b}\left[1-e^{-bt}\right] \text{ For oil systems} \qquad 10$$

Similarly, Gas production: $G_p = \frac{q_i}{b}\left(1-e^{-bt}\right)$ For Gas systems

Oil-Reserves Initially in Place (N) Model Postulation for Projectile Oil Flow Fig 1

$$\begin{bmatrix}\text{Actual Oil Reserves}\\ \text{Initially in Place}\end{bmatrix} = \begin{bmatrix}\text{Area of the}\\ \text{Trapezium ABDO}\end{bmatrix}$$

$$[\text{Area ABDO}] = \frac{q_i}{2}[(t_2-t_1)+(t_f-t_o)]$$

$$N = \frac{q_i}{2}[(t_2-t_1)+(t_f-t_o)] \qquad 11$$

Similarly, Gas Reserves Initially in place: $G = \frac{q_i}{2}[(t_2-t_1)+(t_f-t_o)]$

Equation 11 is the actual stock tank oil initially in place (N). This is very possible since hydrocarbon production is the product of the flow rate, q and time, t (N = q * t).

Postulation of the Parabolic Oil and Gas Flow Models

Data Type – II and Type III: Considerations

An oilfield must contain a reserve initially in place (N), which reduces per unit time during production operations. A reserve must decline right from the initial stage during production in a parabolic dome-shape (Figure 4a) or single-apex shape (Figure 4b). The flow rate (q) of oil or gas stream production continues to change from time, t_o to time, t_1, so that time, t_f could be estimated by extending Pt-X to Pt-y at time-t_f. The actual change in a production rate per unit time is $dq \propto q^n dt$ and the constant of proportionality is "b" or it is the product of the decline rate constant, b and flow rate raised to power-n ($-bq^n$). The cumulative hydrocarbons production (N_p) per unit time would be reduced from the maximum at bubble point (transition state) value to minimum at a given time. The quantity of the reserves remaining in the reservoir is N_f at time t_f.

Hydrocarbons Production Models

Basically 3 types of decline trends in oil and gas recovery were used 1st order equation where n = 1, 2nd order equation where n = 2 and fraction order equation where n <1 or n <2. The general equation for natural production of an oilfield reserves is the product of the rate-constant and the actual rate raised to power-n. This is given by parabolic flow regime (Eqn12):

$$\begin{bmatrix}\text{Actual Change}\\ \text{in Production}\\ \text{Rate with Time}\end{bmatrix} = \begin{bmatrix}\text{A Decline Rate}\\ \text{Constant}\end{bmatrix}\begin{bmatrix}\text{Actual Rate in}\\ \text{n-order}\end{bmatrix}$$

$$\frac{dq}{dt} = -bq^n \qquad 12$$

Using the curve in Figure 1 and Eqn13, the actual oil or gas production rate in a given time is postulated as follows:

When n=1: 1st Order Decline Rate Parabolic Flow

$$\int_{q_i}^{q}\frac{dq}{q} = -b\int_{o}^{t} dt \qquad 13$$

Solving Eqn13 gives the governing equation, Eqn14

$$\ln q - \ln q_i = -bt \quad \text{and} \quad q = q_i e^{-bt} \qquad 14$$

The governing equation, Eqn14 is used to obtain hydrocarbons production rate, q and the rate-constant "b". To obtain the rate, q, remove the log in Eqn14 and rearranging gives Eqn15. To estimate the rate constant (b), the governing equation is applied at point-A, point-B and point-C of Figure 3 below generating 3 equations and simultaneously each pair is solved for "b".

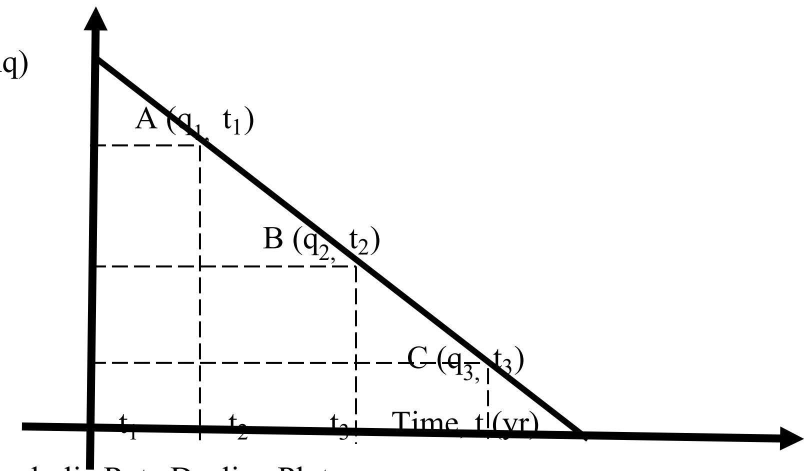


Fig 3 Parabolic Rate Decline Plot

At point-A and point-B, the rate decline equations are Eqn16 and Eqn17 Solving these simultaneously give Eqn18

$$\ln q_1 - \ln q_i = -bt_1 \qquad 16$$

$$\underline{-(\ln q_2 - \ln q_i = -bt_2)} \qquad 17$$

$$(16)-(17): \quad \ln\frac{q_1}{q_2} = b(t_2 - t_1) \quad \text{or} \quad b = \frac{\ln(q_1/q_2)}{t_2 - t_1} \qquad 18$$

If $b_1 = b_2 = b_3 = \ldots = b_n$ it implies uniform decline and n=1, so the equation $b = \frac{\ln(q_1/q_2)}{t_2 - t_1}$, would be used in projecting the flow rate, q for a given time, t. That is q_1 at t_1, q_2 at t_2, q_3 at t_3, ... q_n at t_n using Eqn14

Projected Cumulative Reserves Production Models

$$G_p = \frac{q_i}{b}\left[1 - e^{-bt}\right] \quad \text{For gas systems} \qquad 19$$

$$N_p = \frac{q_i}{b}\left[1 - e^{-bt}\right] \quad \text{For oil systems} \qquad 20$$

When n=2: 2^{nd} Order Decline Rate Parabolic Flow

$$\int_{q_i}^{q} \frac{dq}{q^2} = -b\int_{o}^{t} dt \qquad 21$$

Solving (integrating) Eqn21 gives Eqn22

$$\frac{1}{q_i} - \frac{1}{q} = -bt \qquad 22$$

Multiply LHS of Equ22 by q_i and rearrange gives Equ23, the governing equation.

$$q = \frac{q_i}{(1 + bt)} \qquad 23$$

The governing equation (Eqn22) is the 2^{nd} order decline rate used to obtain hydrocarbons production rate, q and the rate decline-constant, b when the decline exponent is two (n=2). To estimate the rate constant, "b" the governing equation is applied at point-A, point-B and point-C of Figure 4 below generating 3 equations and simultaneously each pair is solved for "b" or just re-arranged making b the subject of the formular ($q = q_i - bqt$). Plotting q vs t, the slope is -bq and intercept is q_i.

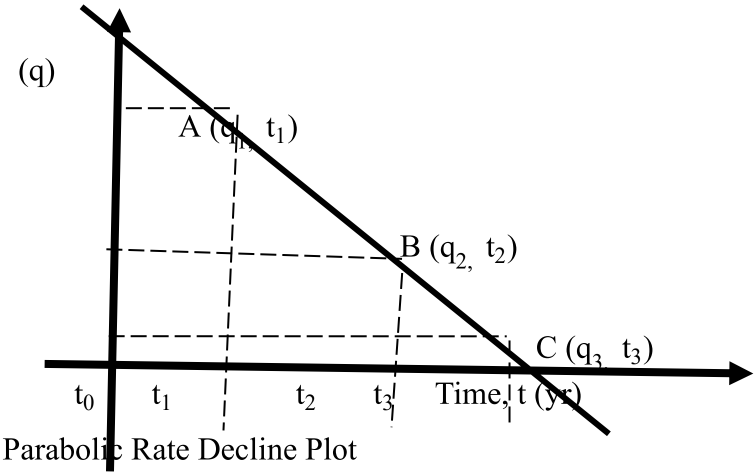


Fig 4 Parabolic Rate Decline Plot

At point-A and point-B, the rate decline equations are Eqn24 and Eqn25. Solving these simultaneously give Eqn26.

$$q_1 = q_i - bq_1t_1 \qquad 24$$

$$(24) - (25): \quad -(q_2 = q_i - bq_2t_2) \qquad 25$$

$$q_1 - q_2 = b\left(q_2t_2 - q_1t_1\right)$$

$$b = \frac{q_1 - q_2}{q_2t_2 - q_1t_1} \quad \text{or} \quad \frac{q_i - q}{q\,t} \qquad 26$$

If $b_1 = b_2 = b_3 = \ldots = b_n$, indicating a uniform decline rate when, n=2, so the equation, $b = \frac{q_1 - q_2}{q_2t_2 - q_1t_1}$ or $\frac{q_i - q}{q\,t}$ would be used in projecting the flow rate, q for a given time, t. That is q_1 at t_1, q_2 at t_2, q_3 at t_3, ... q_n at t_n using Eqn23.

Projected Hydrocarbons Production Models

$$G_p = \frac{[q_i - q]}{b} \quad \text{or} \quad qt \qquad \text{For Gas} \qquad 27$$

$$N_p = \frac{[q_i - q]}{b} \quad \text{or} \quad qt \qquad \text{For Oil} \qquad 28$$

When $n < 1$ or $1 < n < 2$ (Fraction). The value of, $b_1 \neq b_2 \neq b_3 \neq \ldots \ldots \neq b_n$, it indicates non-uniform decline rate. In this case an average decline would be used or the decline rate would be estimated at each point, in the projected flow rate, q within the given time, t. This means b_1 at t_1 is calculated and used for G_{p1} or N_{p1}, b_2 at t_2 is equally calculated and used for G_{p2} or N_{p2}, b_3 at t_3 is also calculated and used for G_{p3} or N_{p3}, and so on to b_n at t_n is used for G_{pn} or N_{pn},

$$q = \frac{q_i}{1 + bt} \qquad (i = 0, 1, 2, 3, , \ldots ., n) \qquad 29$$

$$b \approx \sum_i^n \left[\frac{q_i - q_{i+1}}{q_{i+1} * t_{i+1}}\right] \text{ Meaning: } b \approx \frac{1}{n}\left[\frac{q_i - q_1}{q_1\, t_1} + \frac{q_i - q_2}{q_2\, t_2} + \frac{q_i - q_3}{q_3\, t_3} + \ldots + \frac{q_i - q_n}{q_n\, t_n}\right] \quad 30$$

Projected Hydrocarbons Production

$$G_p = \frac{q_i - q}{b} \quad \text{or} \quad qt \qquad \text{and} \qquad N_p = \frac{q_i - q}{b} \quad \text{or} \quad qt \qquad 31$$

Cumulative Hydrocarbons Production Model

The general equation for natural production of an oilfield reserves is the product of the hydrocarbons flow rate and the actual time elapsed. This is given by parabolic flow regime (Eqn32 and Eqn33):

$$\begin{bmatrix}\text{Actual Cumulative} \\ \text{Hydrocarbons} \\ \text{Produced With Time}\end{bmatrix} = \begin{bmatrix}\text{Actual Production} \\ \text{Rate Per Unit Time}\end{bmatrix}\begin{bmatrix}\text{Actual Time} \\ \text{That Elapsed}\end{bmatrix}$$

$$G_p = qdt \qquad 32$$

Similarly: $$N_p = qdt \qquad 33$$

Eqn32 or Eqn33, the actual gas or oil cumulative production at a given time is postulated as follows:

$$G_p = \int_o^t qdt \quad \text{or} \quad N_p = \int_o^t qdt \qquad 34$$

But $q = q_i e^{-bt}$ substituting this in Eqn34 gives Eqn35, the cumulative gas or oil production

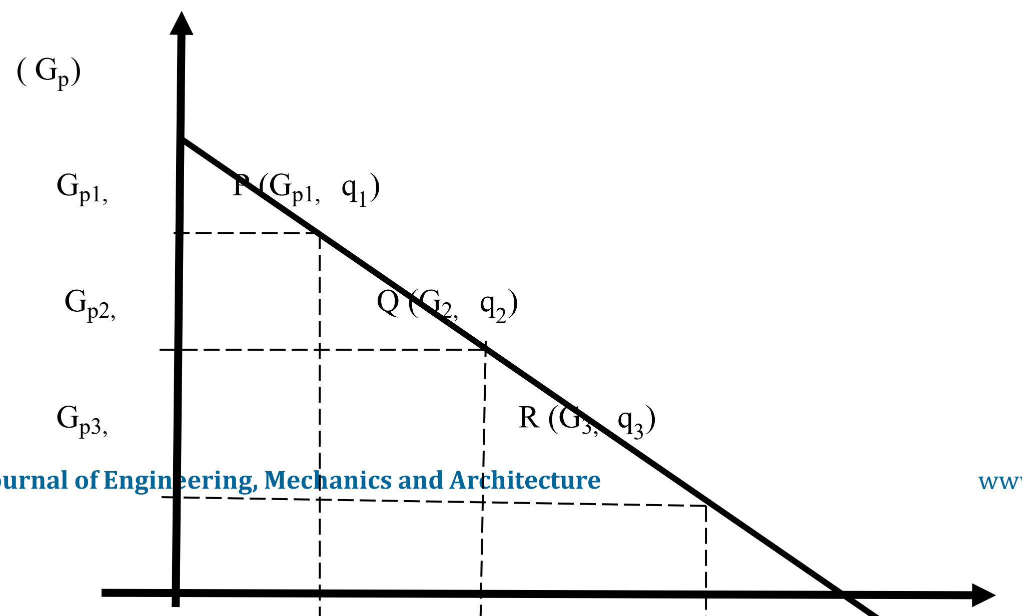
$$G_p = \int_o^t q_i e^{-bt} dt \qquad \text{or} \qquad N_p = \int_o^t q_i e^{-bt} dt \qquad 35$$

Solving Eqn35 gives Eqn36, the governing equation for gas or oil cumulative production.

$$G_p = \frac{q_i}{b}\left[1 - e^{-bt}\right] \qquad \text{or} \qquad N_p = \frac{q_i}{b}\left[1 - e^{-bt}\right] \qquad 36$$

Eqn36 is the governing equation, used for estimating the gas cumulative production value, G_p and oil cumulative production value, N_p. But, $q = q_i e^{-bt}$, substituting this in $N_p = \int_o^t q_i e^{-bt} dt$ *(Eqn36) and re-arrange gives Eqn37. $q - q_i = -bG_p$ Similarly: $q - q_i = -bN_p$ 37

To estimate the rate constant, "b" using cumulative gas or oil production, Eqn37 is applied at point-P, point-Q and point-R of Figure 5 below, which shows a plot of cumulative hydrocarbons production against the rate. The plot generated 3 equations and two of the equations were solved simultaneously for "b" as follows:



q_1 q_2 q_3 Rate, q (stb/t)

Fig 6 Cumulative Hydrocarbons Production Plot

At point-P and point-Q, the rate decline equations are Eqn38 and Eqn39. Solving these simultaneously give Eqn40.

$$q_1 - q_i = -bGp_1 \qquad 38$$

$$(3.8) - (3.9): \underline{-(q_2 - q_i = -bG_{p2})} \qquad 39$$

$$q_1 - q_2 = b(G_2 - G_1)$$

$$b = \frac{q_1 - q_2}{G_2 - G_1} \qquad \text{For Gas} \qquad 40$$

$$\text{Similarly: } b = \frac{q_1 - q_2}{N_{p2} - N_{p1}} \qquad \text{For Oil} \qquad 41$$

If $b_1 = b_2 = b_3 = \ldots\ldots\ldots = b_n$ it implies uniform decline, so the equation $b = \frac{q_1 - q_2}{G_2 - G_1}$ could be used in projecting the hydrocarbons production, G_p for a give time, t. This means that the projected gas produced is done as follows: G_{p1} at t_1 using b_1, G_{p2} at t_2 using b_2, q_3 at t_3, using b_3, ...q_n at $t_n$using b_n in gas production and for oil production respectively.

Result

Figure 7 shows projectile flow and fig 8 shows Parabolic Flow Types: Short Transient with short or no Transient or Transition Times.

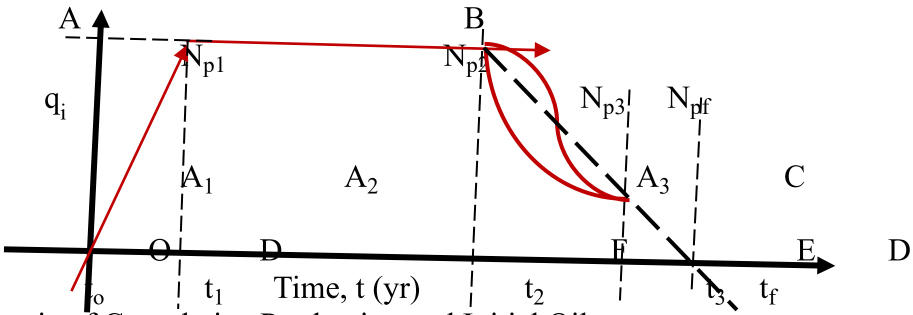


Fig 7 Schematic of Cumulative Production and Initial Oil

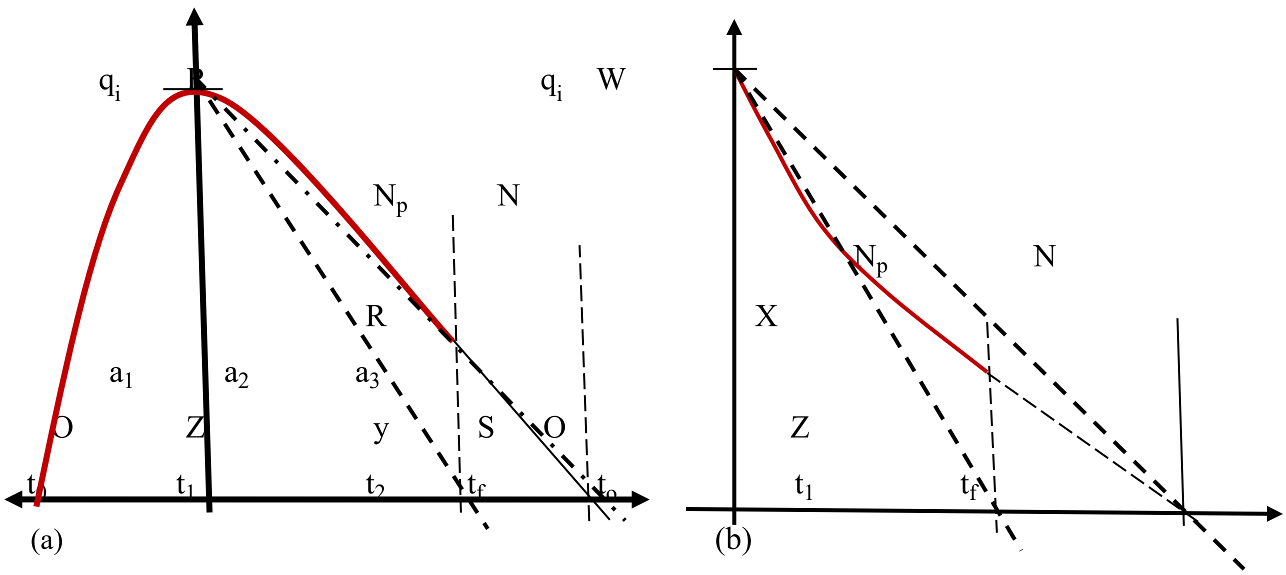


Fig 8 Schematic of Oil or Gas in Parabolic Flow Regime

Table 1: Confirmed Projectile Evaluation Models

Type	Eqn	Model Equation	Remarks
	7	$b = \frac{\ln(q_i/q)}{t - t_i}$ For n=1	
	26	$b = \frac{q_i - q}{q\,t}$ For n=2 or others	
Projectile Gas and Oil Flow Models	3	$N_p = \frac{q_i}{2}[(t_2-t_1)+(t_3-t_o)]$ $G_p = \frac{q_i}{2}[(t_2-t_1)+(t_3-t_0)]$	Models for Estimation of Cumulative Gas and Oil, In MSCF, Fig 1/2
	11	$N = \frac{q_i}{2}[(t_2-t_1)+(t_f-t_o)]$ $G = \frac{q_i}{2}[(t_2-t_1)+(t_f-t_0)]$	Models for Estimating Oil and Gas Initially in Place, SCF, Fig 1.2

Table 2: Models Applications: Confirmed Parabolic Evaluation Models

Type	Eqn	Model Equations	Remarks
Decline Rate for (n=1)	14/18	$b = \frac{\ln(q_1/q_2)}{t_2 - t_1}$ and $q = q_i e^{-bt}$ $G_p = \frac{[q_i - q_{i+1}]}{b_{yr}}$ or $G_p = \frac{q_i}{b}(1-e^{-bt})$ $N_p = \frac{[q_i - q_{i+1}]}{b_{yr}}$ or $N_p = \frac{q_i}{b}(1-e^{-bt})$	Stb/time, t Per time, t Fig 3
Decline Rate for (n=2)	23/26 27/28	$q = \frac{q_i}{1+bt}$ and $b = \frac{q_i - q}{q\,t} = \frac{q_1 - q_2}{q_2 t_2 - q_1 q_1}$ $G_p = \frac{[q_i - q]}{b}$ and $N_p = \frac{[q_i - q]}{b}$ or qt	Per time, t Fig 3
Decline Rate for fractions (n <1) or (n<2)	29 30 3.48 27/28	$q = \frac{q_i}{1+bt}$ (i=0,1, 2, 3, , ..., n) $b \approx \sum_i^n \left[\frac{q_i - q_{i+1}}{q_{i+1} * t_{i+1}}\right]$ Meaning: $b \approx \frac{1}{n}\left[\frac{q_i - q_1}{q_1\,t_1} + \frac{q_i - q_2}{q_2\,t_2} + . \ . \ . \ . + \frac{q_i - q_n}{q_n\,t_n}\right]$ $G_p = \frac{[q_i - q]}{b}$ and $N_p = \frac{[q_i - q]}{b}$ or qt	
For Easy Unit Time Conversion		$e^{-bt} = (1-b)^t$, (Taylor''s Expansion) $(1-b/yr) = (1-b/m)^{12} = (1-b/d)^{365.25}$ $b/yr = 12*b/m = 365.25b/d$	

Discussion

Figure 7 shows a schematic of oil or gas cumulative production and initially in place for projectile gas or oil flow. A reservoir starts from an initial flow rate by building up the internal energy for some time from time "t" _"o" to time ("t" _"1"). This is because the reservoir was fairly saturated, so it failed to attain boundary-dominated flow at the initial state [10]. After which it remained steady for another time ("t" _"1" "-" "t" _"2"), when the external energy went lower it begins to decline at time "t" _"2" down to time "t" _"3" . At this point, the graph was extrapolated time "t" _"f" for estimation of reserves initially in place. Figure 8a/b show schematic of oil and gas cumulative production and initially in place in parabolic flow [11]. In this case a reservoir built-up from the initial stage time "t" _"o" to time "t" _"1" , the transient and transition stage at point – Q, but the flow period was too short. To this effect, steady-state flow (called the plateau) was not observed in the curve at time," " 〖" t" 〗 _"1" instead rate decline state began from time," " "t" _"1" to time "t" _"2" at point R (fig 8a) [12]. The dome shape of Fig 8a indicates a parabolic flow rate from lowest at point-P to a maximum point –P and declines to abandonment at point – R. The curve was extrapolated from point- R at "t" _"1" to point –S at "t" _"f" , for estimation of oil-reserve initially in place by extension of curve-PR at point-R to S in time-"t" _"f" . In the case of Figure 8b the reservoir pressure was just slightly above the bubble point or at bubble point pressure [13]. The implication of this case is that decline starts right from the early age of production at point–W to point-X. The curve was extrapolated from point- X to point–y, for estimation of oil initially in place. The early production data were projected to induced abandonment periods for estimating the cumulative reserves production and initially in place [14]. The outstanding advantages of the decline stage models include: Prediction of the daily oil production rate and cumulative recovery in a given period. This enables the operator to equally predict the abandonment period and the cumulative recovery value. Good prediction of the reserves in place when the decline rate stage is converted to a pressure-dominated flow stream [15].

Conclusion

Mathematical or evaluation models were derived for studying reservoir fluid depletion from the peak value to an economic value called abandonment. Estimation of oil and/or gas cumulative production and initially in place was done using production decline rate trend models. Decline rate trend analysis showed two types of flow regimes, namely projectile-dominated flow regimes attributed to little or no boundary conditions effects and Parabolic flow regimes whose principal mechanism is due to boundary conditions effects. The projectile-dominated flow models were mainly used and generated curves for predicting hydrocarbon production performance. It was observed that above bubble-point pressure, projectile-dominated flow (model I) should be used, but when reservoir pressure is close to or at bubble-point pressure, a parabolic-dominated flow (model II) should be used. The extrapolation of the curve from the decline point to the economic rate point on the t-axis at "t" _"f" (Fig 7) gave the total reserves in place. Projectile models percentage accuracy for gas fields ranged from 99.86% and above, while the percentage accuracy for oil ranged from 98.64% to 99.98%. This research work contributes in two folds to petroleum engineering, namely technologically and economically.

Technologically:

High accuracy future reserves ("N" _"p" " & N") prediction, in long or short field history

Tactfully used in appraisal wells to study initial state of a reservoir and decline rate trend.

Equally tactfully used in an induced hydrocarbon operation (pressure maintenance) to improve the pressure management in a reservoir, because when a reservoir pressure is maintained, it prevents critical depletion and the recovery is high.

The outstanding advantage of this research work is the development of a prosy model that can predict a production trend in a reservoir. A prosy model that shows the production trend graphically.

Graphical solutions enhance the plan for reservoir pressure management or re-pressurisation

Economically

It defines total oil and/or gas recovery for a given time (Recovery factor).

It encourages operators to stay in the business and manage production to normal abandonment.

The hidden details for the Operator and the host to formulate or amend the oil field contractual agreement were clearly worked or defined. Such details include total recoverable reserves and the time to be used. This enhances the decision for the investment bills (CAPET and APEX).

The disadvantage is that the models do not take care of pressure drawdown, so the projected rate decline trend or cumulative hydrocarbons production trend depend on pressure sustainability program by the operator.

Recommendations

Field operators should maintain accurate and consistent production records as the reliability of decline curve analysis depends heavily on the quality of production data available.

Operators should consider combining results from this method with other estimation methods for informed decision-making.

Further study is recommended to ascertain pressure fluctuations and their influence on evaluation models for possible augmentation.

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