

## **Investigation of Waterflooding in an Oil Reservoir by Streamline Simulation**

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**Abstract:** *Perhaps Waterflooding is the most popular method of secondary recovery to pressure stability and support a higher withdrawal rate but the large of produced water in waterflooding of offshore oil reservoir is one of the most important oil industry problems.*

*So determining of optimized injection rate for injection wells is one of the important factors in a successful waterflooding.*

*Like using conventional finite difference method, developments of streamline methods lets streamline based reservoir simulation has a lot of effects on solving complex geologically large and heterogeneous reservoir and also streamline simulation is explained in the text of producer and injector usage, relationship between injectors and producers and well allocation factors. So using of streamline simulators is a suitable method to manage waterflooding in oil reservoir. The purpose of this paper is to explain the usage of streamline method for a waterflooding simulation in an Iranian offshore oil reservoir that leads to optimization of water injection rate for decreasing the produced water from reservoir.*

**Keywords:** *Streamline simulation, finite difference method, waterflooding, optimization, oil reservoir.*

### **1. Introduction**

The most important aim of all reservoir simulation methods is to predict flow performance of oil recovery processes. If we have more realism in our reservoir models and our reservoir simulators, we will have more accuracy in our answers. One of its goals is improvement of the simulation methods efficiency and it is one of the streamline method goals. Muskat[1-2], Fay and Prats[3], Higgins and Leighton[4-5] have worked on the application of streamlines and streamtubes for modeling displacements in heterogeneous media and some main contributions have been explained by Martin and Wegner[6], Bommer and Schechter[7], Lake et al.,[8] and Emanuel et al.[9]. In other words, every streamline is as the center of streamtube which its volume is known but the boundaries aren't specified [10]. In compare with conventional methods, streamline methods are faster and have more accuracy while they use standard computer resources to model field scale displacement of large method. King and Datta Gupta [11] worked on the differences between streamline and streamtube based on simulation. Streamline simulation helps us with solving large complex geologically and heterogeneous system [12, 13, 14, 15, 16] and also has been used successfully for waterflooding studies, well allocation factors, pore volumes and injection efficiency [17]. The aim of this paper is to investigate and simulate waterflooding in an oil reservoir by using streamline simulation. In this investigation, waterflooding process has been

simulated with streamline methods at first and then the results have been compared with a commercial finite difference simulation [18].

## **Methodology**

This paper used a numerical simulation research design utilizing streamline simulation to maximize subsurface fluid flow towards the producer wellbore in an offshore oil reservoir with significant heterogeneity, improving overall waterflooding efficiency while cutting down on excess produced water. Streamline simulation features were analyzed relative to a conventional finite difference grid under equivalent reservoir conditions with a comparative modeling approach. The reservoir model used was 3D grid size of  $35 \times 35 \times 5$  with a total number of active grid blocks equal to over whole heterogeneous formation with different permeability and porosity distributions replicated across the simulated volume in order. A total of twelve production wells and three vertical water injection wells were modeled with constant water injection rates to maintain reservoir pressure and improve oil recovery during the simulation. Mud consolidation: Governing equations of multiphase incompressible flow were based on the Implicit Pressure Explicit Saturation (IMPES) approach and streamline methodology, which made it possible to transform the 3D transport equations into for a number of independent travelling 1-D flow equations directed along streamlines, thus making computations more manageable in terms of run time while reducing numerical dispersion. The simulations were performed for a ten-year production period (2010–2020), and at every time step, reservoir pressure, oil production rate, water production rate, cumulative water injection, water cut and injector–producer connectivity are recorded. Flow distribution between injection and production wells along with inefficient injection patterns were further quantified using allocation factors derived from streamline analysis and time-of-flight. In accordance with these connectivity analyses, injection rates were optimized by redistributing water injection among the three injectors in order to enhance sweep efficiency and mitigate water breakthrough. The evaluation of the simulation results prior to and post optimization was performed by selecting critical field metrics, including water cut in an oilfield, cumulative water production, reservoir pressure maintenance & oil production performance. The proposed simulation methodology integrates a three-dimensional geocellular model, reservoir simulation and proper orthogonal decomposition (POD) to offer a robust test bed for assessing waterflooding strategies and enabling data-driven optimization of the coupled petroleum reservoir management problem in complex heterogeneous settings.

## **Result and Discussion**

### **2. The difference between streamline method and finite difference method**

There is a pressure solving section and a saturation section in conventional finite difference simulation. In finite difference method, at first we try to solve the pressure and then compute flow according to the distribution of pressure but flow transports from grid cell to grid cell while in streamline simulation method fluids are transported among streamlines and it's illustrated in Fig. 1.

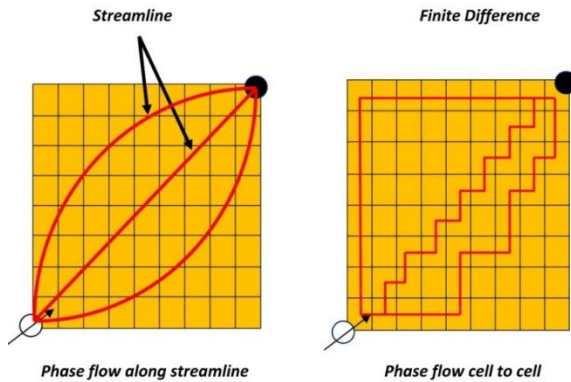


Fig.1.The Difference between finite difference model and streamline model for transporting fluid.

Because of the non-linear transport in finite difference method, this method is sensitive to grid block orientation and their sizes and affects on finite difference simulation time steps. In both of streamline and finite difference method the pressure equation is solved on an underlying grid block, after that streamlines are made orthogonal to pressure and they are tangent to velocity field in the time step. Fig.2.

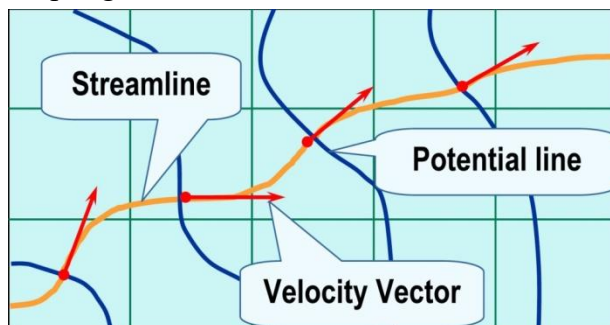


Fig.2.Streamline is tangent to velocity and orthogonal to the pressure.

So a natural transport net is computed and the field is transported among streamline to follow the movement of oil/water/gas in reservoir. Thus streamlines have a natural advantage because the fluid moves in the line of the pressure gradient among the streamlines and it doesn't move between grid blocks and they are shown in Fig. 1. So all of them the cause to a great stability, large time steps, less sensitive to the size of grid blocks and grid orientation [19]. Important idea of the streamline method is to break the governing equation of fluid movement to the full 3D problem into multiple 1D problem solves along streamline [20-16]. So the streamline decreases the numerical dispersion in comparison with conventional finite difference method.

### 3. Mathematical formula

#### 3.1. Governing IMPES Equation

In streamline simulation reservoir dominated flow is based on the IMPES (implicit pressure explicit saturation) and this fact means that the pressure field will be solved at first and then it happens in saturation distribution and these pressure are obtained as in any other conventional finite difference based on reservoir flow simulators. According to an assumption, the capillary pressure is ignored. The pressure equation for multiphase and incompressible flow in porous media is defining [21]:

$$\nabla \cdot \vec{k} \cdot (\lambda_t \nabla P + \lambda_g \nabla D) = 0 \quad (1)$$

Total mobility and total gravity mobility,  $\lambda_t$  and  $\lambda_g$  are:

$$\lambda_t = \sum_{j=1}^{n_p} \frac{k_{rj}}{u_j}, \quad \lambda_g = \sum_{j=1}^{n_p} \frac{k_{rj} \rho_j g}{u_j} \quad (2)$$

The governing saturation eq, of an individual phase gets[22]:

$$\phi \frac{\partial S_j}{\partial t} + \vec{u}_t \cdot \nabla f_j + \nabla \cdot \vec{G}_j = 0. (3)$$

The fractional flow term,  $f_j$  is:

$$f_i = \frac{k_{ri}/\mu_j}{\sum_{i=1}^{n_p} k_{ri}/\mu_i}, (4)$$

Batycky et al., [12-13] and Thiele et al.[23]introduced the total gravity fractional flow term by this formula:

$$\vec{G}_j = \vec{K} \cdot g \nabla D f_j \sum_{j=1}^{n_p} k_{rj}/\mu_i (p_i - p_j) (5)$$

In compare with conventional finite difference method, in the streamline method eq.3 will be decoupled into multiple 1D equations and solving multiple 1D equations is much faster and more accurate.

### 3.2. Coordinating transformation along streamlines:

As the streamlines are traced from injection wells to production wells, we explain the Time of Flight (TOF)[11-24]. Time of flight is required time to reach the distance along streamline.TOF is based on the velocity field along streamline which is defined as:

$$\tau = \int_0^s \frac{\phi}{u_t(\zeta)} d\zeta (6)$$

To explain the coordinate transform we use eq.5 as:

$$\frac{\partial \tau}{\partial s} = \frac{\phi}{|u_t|} (7)$$

And also can be written as:

$$|u_t| \frac{\partial}{\partial s} \equiv \vec{u}_t \cdot \nabla = \phi \frac{\partial}{\partial \tau} (8)$$

We should substitute eq.8 into eq.3 [21]:

$$\frac{\partial S_j}{\partial \tau} + \frac{\partial f_j}{\partial \tau} + \frac{1}{\phi} \nabla \cdot \vec{G}_j = 0. (9)$$

Eq.9 is one of the important pseudo-1D phase material balance equation for transforming along a streamline. We call it pseudo-1D because the gravity term is typically not aligned along the direction of a streamline. Now we can use the operator splitting concept [13,14,15]. Eq.9 is split into two parts and solving the first one ends to the initial condition for the second one [12,13,15].explained that eq.10 is a convective step along streamline that consists of well boundary conditions , is taken to contract an intermediate saturation dispersion  $S_j^c$  and eq.11 is a gravity step along gravity lines. We consider  $S_j^c$  as the initial condition for constructing  $S_j$  in the next time step[21]. For having a simple assumption we suppose that the gravity lines are aligned in the Z coordinate direction .(Fig. 3).

$$\frac{\partial S_j^c}{\partial \tau} + \frac{\partial f_j}{\partial \tau} = 0 (10)$$

$$\frac{\partial S_j}{\partial \tau} + \frac{g}{\phi} \frac{\partial G_j}{\partial z} = 0 (11)$$

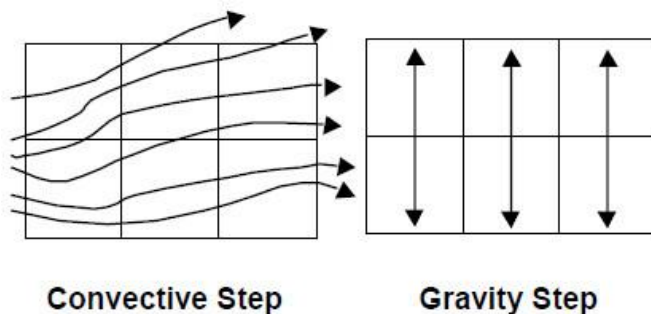


Fig.3.convective step is among streamline and gravity step is along gravity lines (Thiele 2003).

#### 4. Time stepping

When we use mapping numerically solutions we will have an important idea that says the streamline paths change in time because of changing total mobility field. So pressure field is updated according to these changes. For moving forward in time from  $t^n$  to  $t^{n+1}=t^n+\Delta t^{n+1}$  we use the following algorithm [16-17-20-21-25]and visualize in Fig.4.

1. Once the grid has been populated and the initial and boundary conditions are defined, the pressure solution is found for a time step. Pressure solution is found for a time step. Pressure field is generated using a finite difference method.
2. The velocity is computed at grid block faces.
3. The streamlines are generated.
4. The time of flight is computed along streamlines.
5. The saturations are mapped onto streamlines.
6. The saturation equation is solved for each streamlines, using a 1D numerical solution.
7. The gravity segregation is solved, treating as necessary.
8. Saturations are mapped back to the grid.
9. The pressure solution is found for the next time step.
10. Return to step 2.

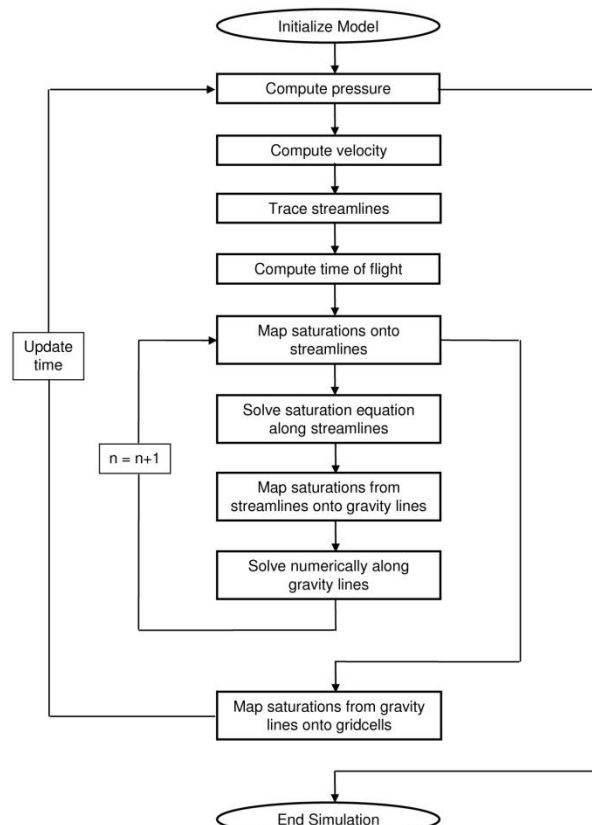


Fig.4. Streamline simulation flow chart (Lolamari et al., 2000).

#### 5. Reservoir model

We study on an offshore oil reservoir that has a large amount of produced water to optimize produced water. Our reservoir was up scaled to 35\*35\*5 and contained 6125 active grid block. On

this grid network permeability was ranged from 0.109 mD( $10.76e-5 \mu\text{m}^2$ ) to 54.4 mD( $5.35e-2 \mu\text{m}^2$ ) and porosity was ranged from 1.4% to 7.3%. Other factors of our reservoir are illustrated in table 1.

Table 1. parameters of reservoir model

Parameters used in the simulations		Values in English units
Reservoir grid		35*35*5
Total Injection rates	18000 STB/day	0.03312 m <sup>3</sup> /s
$\mu_w$	0.31 cp	3.1e-04 Pa.s
$\rho_w$	62.3 b/ft <sup>3</sup>	997.95 Kg/m <sup>3</sup>
$\rho_o$	52.1 b/ft <sup>3</sup>	834.56 Kg/m <sup>3</sup>
$p_i$	3358.86 psi	23145.9 KPa

This reservoir has 12 produced wells with 3 vertical injection wells (I1, I2 and I3) while each of them injects  $0.01104 \text{m}^3/\text{s}$  water into the reservoir. There is an aquifer in the reservoir boundary so we inject water into our reservoir to fix our pressure, increase our production, support a higher drawl rate and distribute the water volume to different areas of the field to achieve more uniform areal converge. At first we simulated waterflooding by streamline simulation and finite difference simulation for ten years (2010 to 2020) then compared their results and optimized reservoir produced water.

Oil saturation by streamline simulation and finite difference simulation is illustrated after ten years of waterflooding simulation in Fig.5 and Fig. 6.

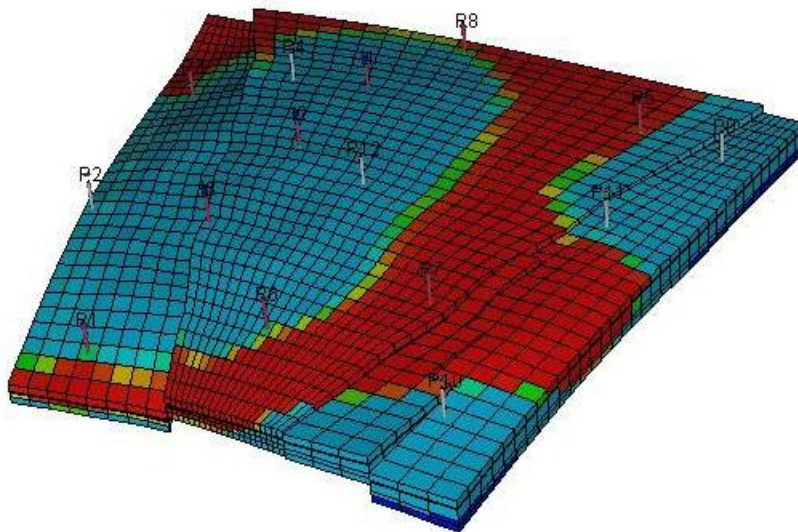


Fig.5. finite difference method.

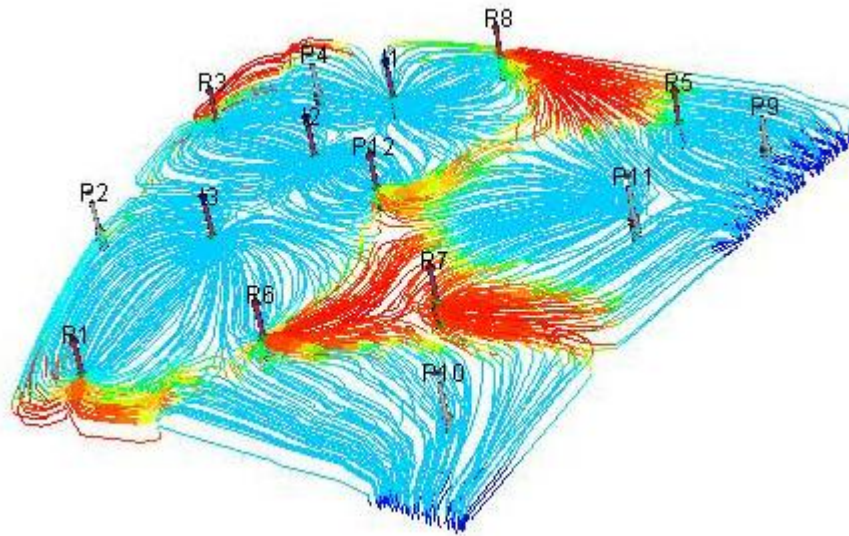


Fig.6. streamline method.

We compute field pressure by streamline simulator and finite difference simulator, and then compare them in Fig. 7. Both simulators use the same methods for solving the pressure equations and we expect to see some results regarding the pressure. But they use different initializing methods and Eclipse uses fully-implicitly method while Frontsim [26] solves the pressure equations explicitly. That's why the calculated pressure is different.

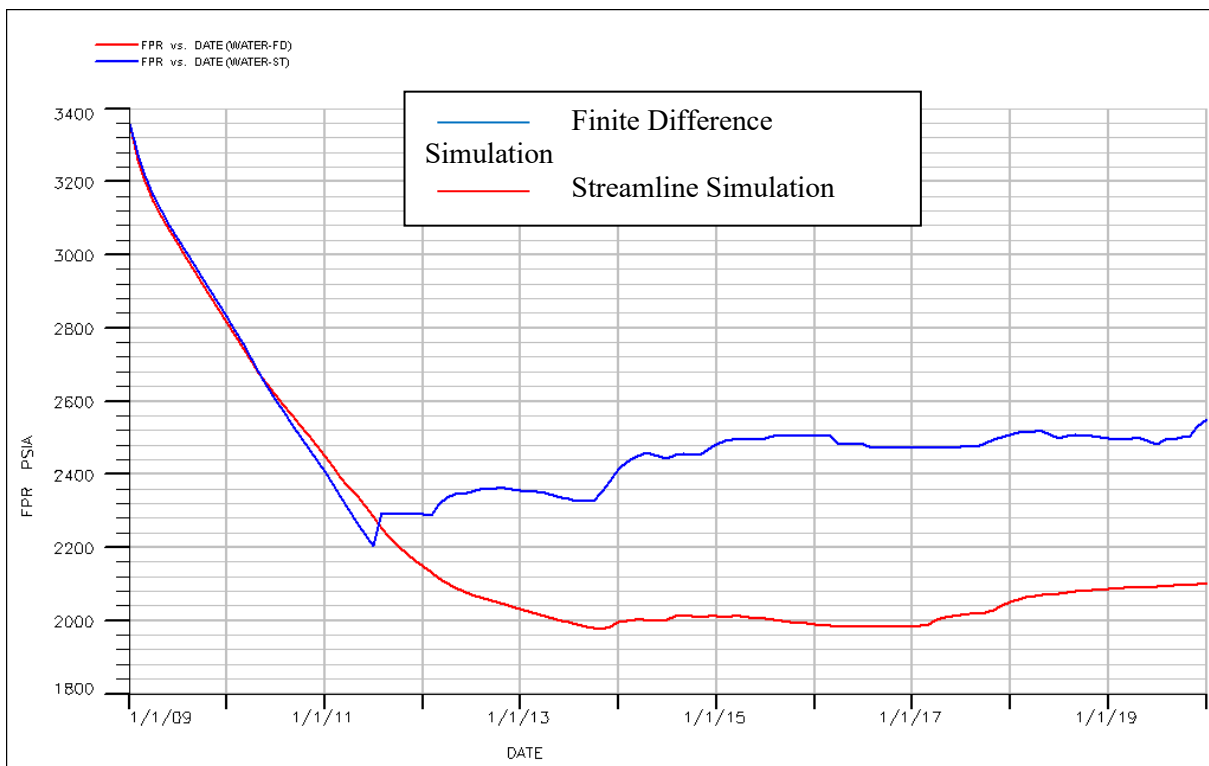


Fig. 7. Comparison of field pressure curves.

Field oil rate production has been illustrated with both of simulators in Fig. 8. Both of them show maximum produced oil during 1 year but oil rate production will be decreased after awhile. Both of them have an excellent connection during 3 years.

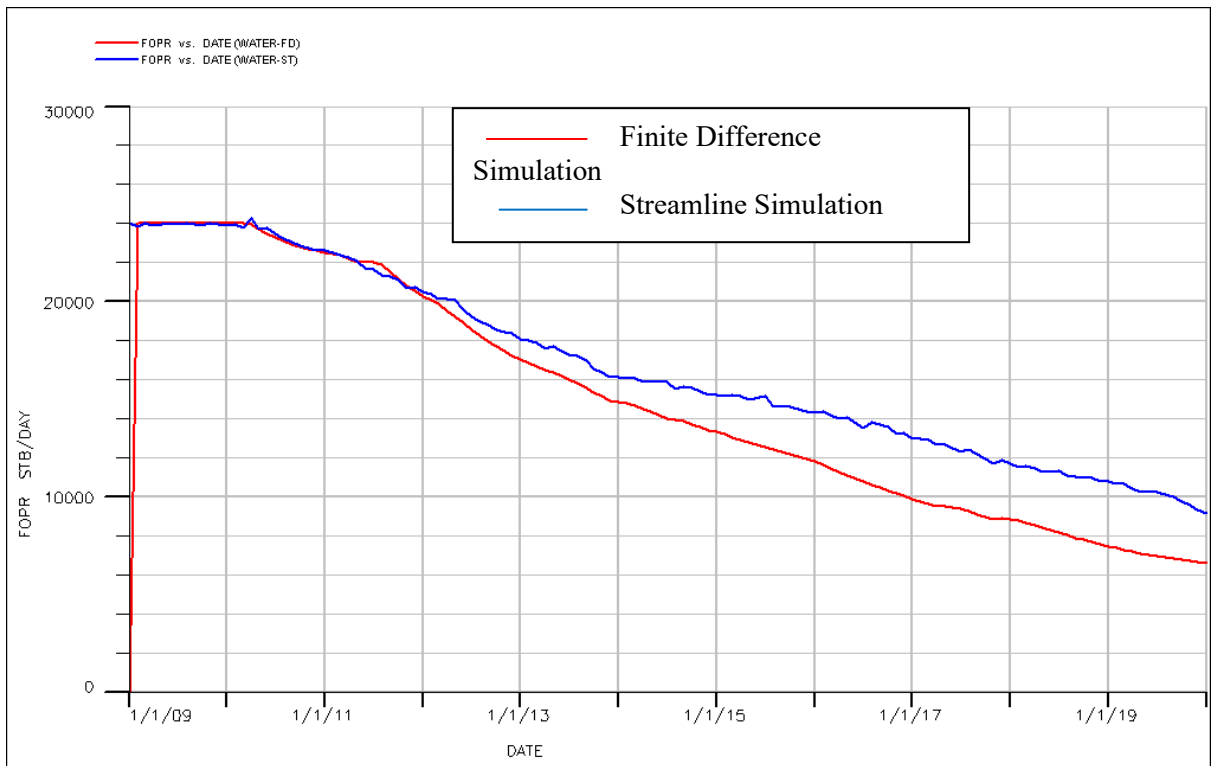


Fig.8. Comparison of field oil production rate.

The difference between field water production rate and field water production total in both of simulators is illustrated in Fig. 9 and Fig.10. In waterflooding of oil reservoir with pushing oil by using water into produced well, water will be produced after awhile and we have an increasing rate of produced water.

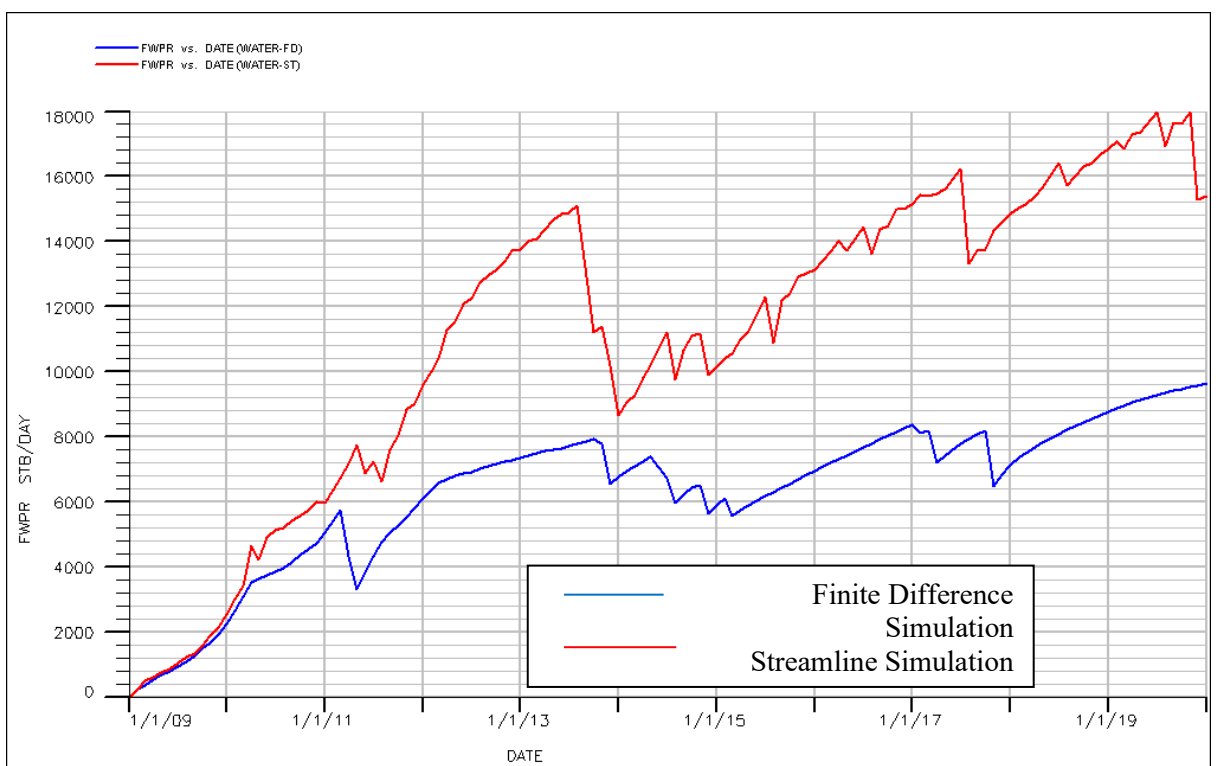


Fig.9. Comparison of field water production rate.

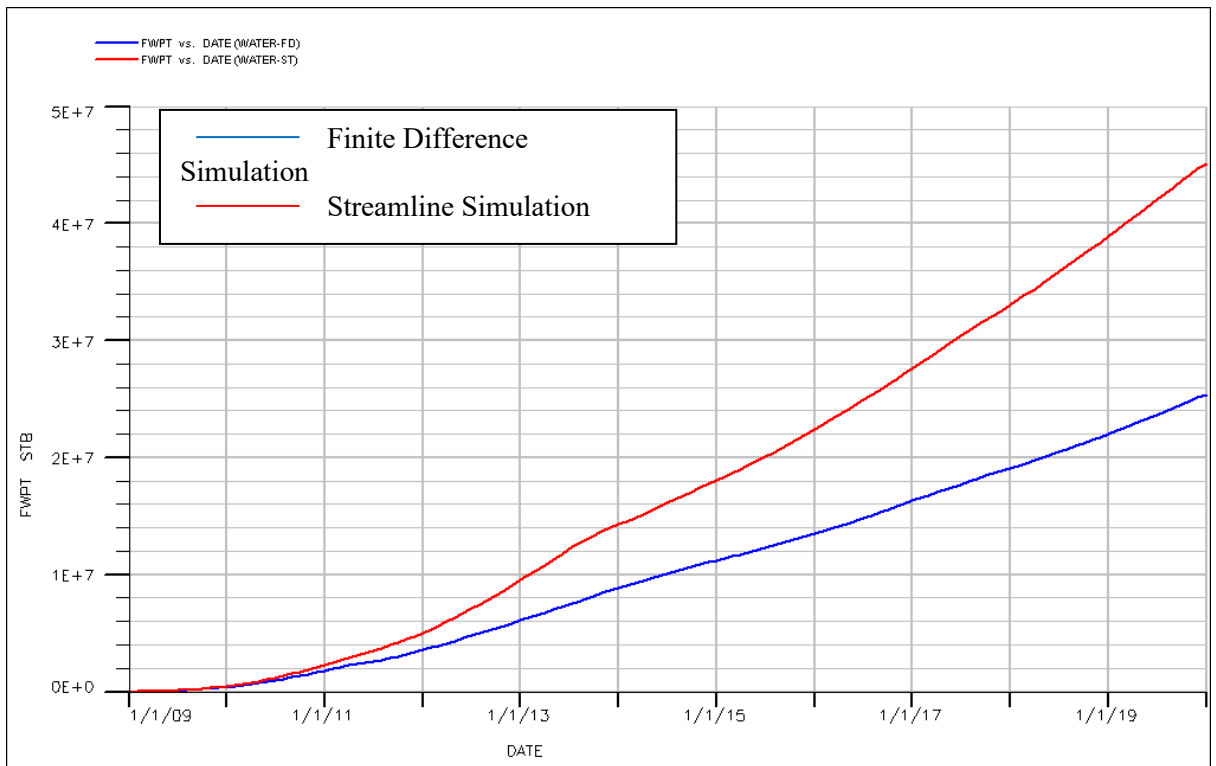


Fig.10. Comparison of field water production total.

We compare the effects of fluid water cut in Fig. 11. The reason of tolerance in Fig. 11 is because of non-stability of solution methods.

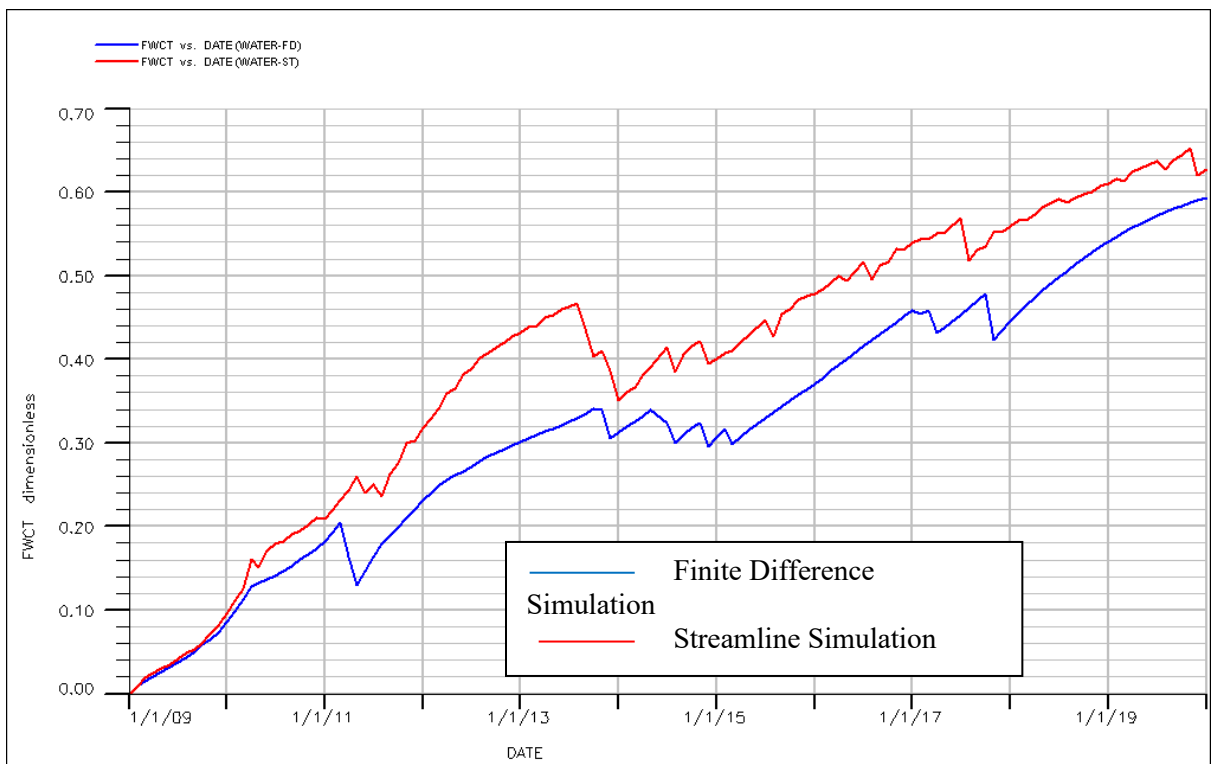


Fig.11. Comparison of field water cut.

The streamline method can manage the reservoir by gathering important data about the relation between injectors and producers and allocation factors [20]. We can achieve this information by using streamline methods but not from conventional finite difference methods. In Fig. 6 we illustrate the relationship between injectors and producers with different colors. Due to the association between streamlines and flow rates, we can calculate allocation factors for injectors,

for example computing factor of the injected fluid to producers is gained. Percentage of flow distributions from injectors to producers are shown in Fig. 12.

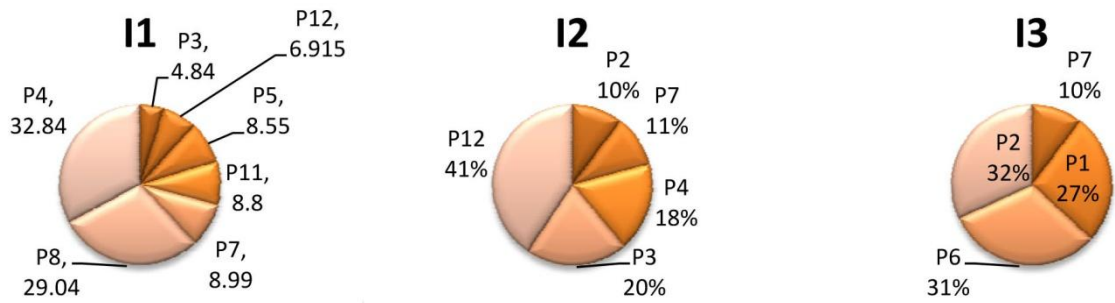


Fig.12. Percentage of flow distributions from injectors to producers.

## 6. Waterflooding optimization

In this paper although we try to concentrate on water injection and oil production, the description of efficiency can be developed for all kinds of injected and produced volumes by applying suitable volumes in the definition of the injection efficiency.

Model of reservoir simulation by streamline method has been illustrated in Fig. 6 and we explained that each well injection injects  $0.01104\text{m}^3/\text{s}$  into reservoir. But with studying their results we understand that the flow is not clearly balanced. What is the injection efficiency of injector I1? From the streamlines, I1 is connected to six producers P3, P4, P5, P7, P8, P11 and P12 and the injection rate I1 is clear. Offset oil production at the producers according to the injection of I1?

The oil is produced by different streamline bundles that start in I1 and end in varies producers, e.g. the oil produced by the red, green, orange, blue, light blue, assure and yellow streamlines. We obtained this data from well allocation factors. The data of streamlines can be illustrated schematically by a flux pattern (3DSL 2005) and it shows the connection between well pairs [10].

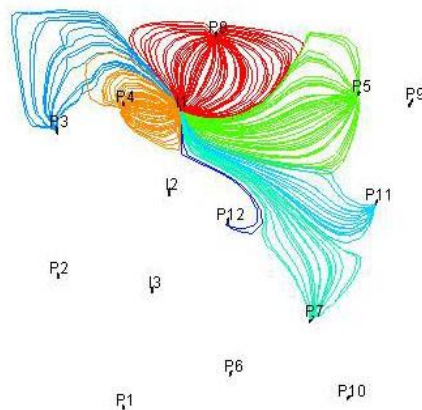


Fig.12.The injection efficiency of injector I1.

Fig. 13 shows that 33% of injected volume goes to producer P4, while just 29% supports P8. The off pattern producer P7 receives 9% of injected volume of I1. The thickness of the lines connecting the wells can be used to represent the strength flux between the pairs [10].

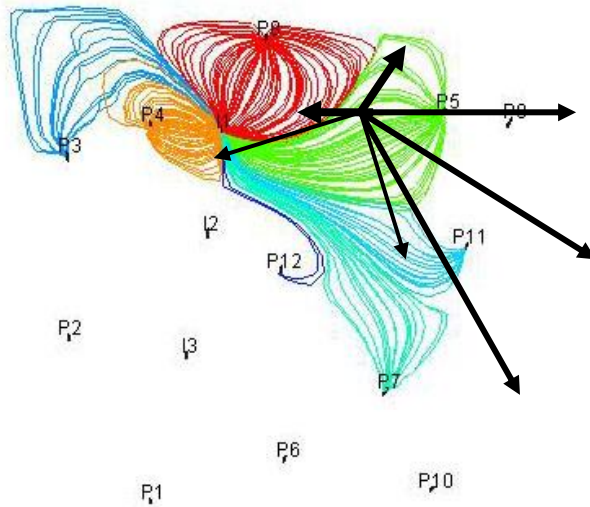


Fig.13.Flux pattern map for injector I1.

As we mentioned before we had 3 injected wells with  $0.01104\text{m}^3/\text{s}$  injection rates and total injection rates  $0.03312\text{m}^3/\text{s}$  and now we understand that the injector I1 has the most effects on the oil recovery in produced wells. So we increased the injection rate of I1 to  $0.02208\text{m}^3/\text{s}$  and reduced the injection rates of I2 and I3 to  $5.52\text{e-}03\text{m}^3/\text{s}$  to decrease the produced water from field. The results of optimization are illustrated in Fig. 14, Fig. 15 and table 2. So we reduced field water cut to 0.49.

Table 2.Comparison between result of optimized and unoptimizedwaterflooding.

Parameters	Unoptimized	Optimized
FWCT	0.62	0.49
FWPT	$83.03\text{m}^3$	$58.31\text{m}^3$

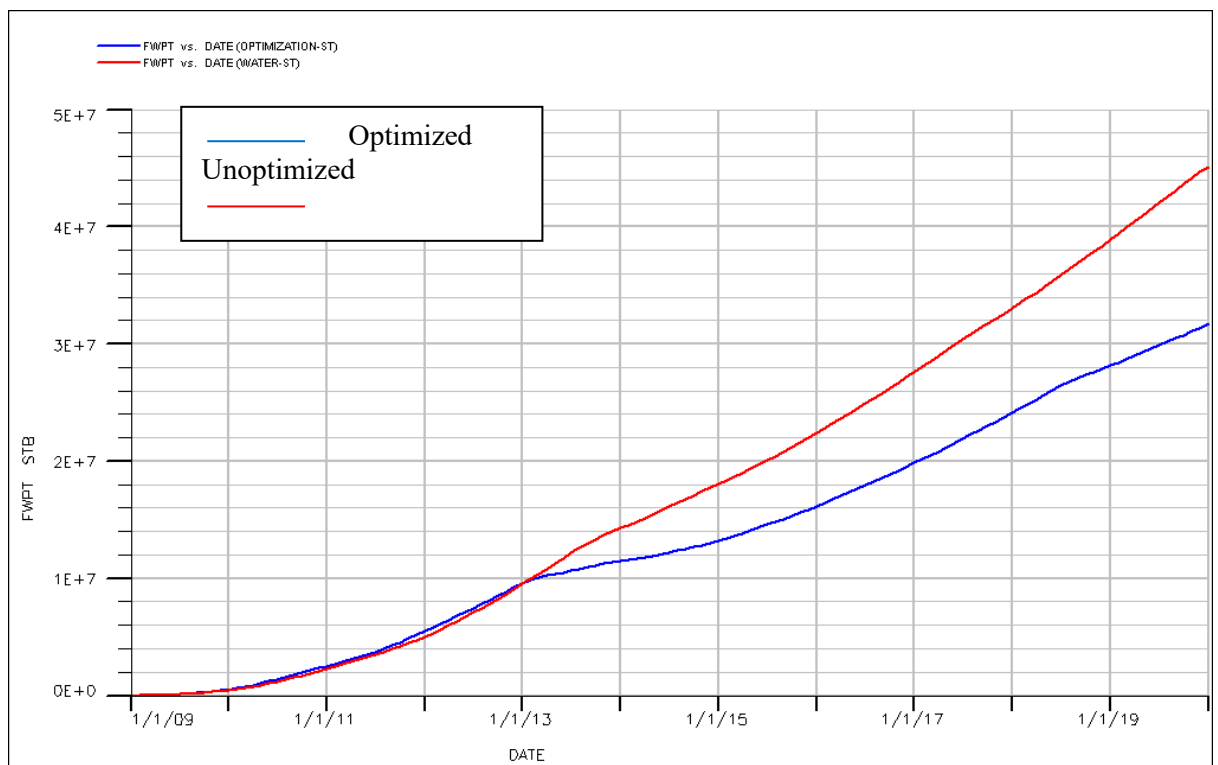


Fig.14.optimized and un-optimized field water production total.

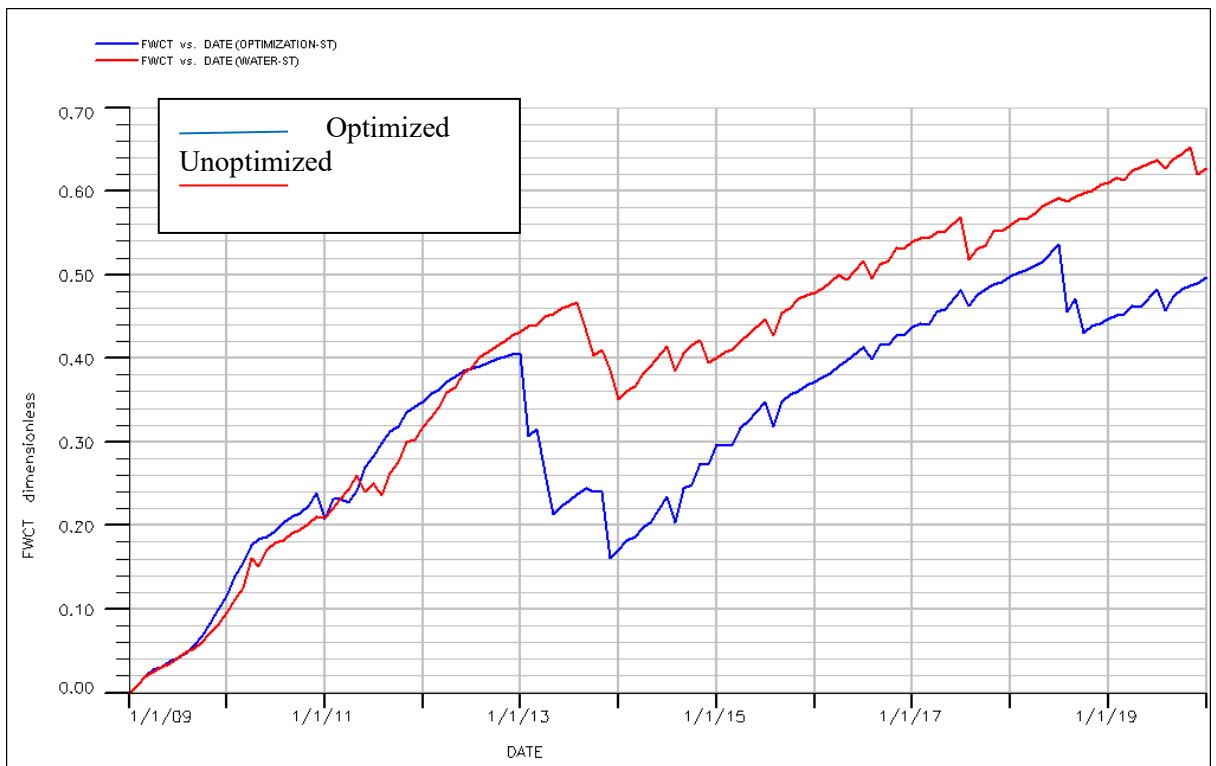


Fig.15. Optimized and un-optimized field water cut.

## 7. Discussion

Simulators based on streamlines and finite difference methods have different results. In streamline simulation, saturation equations are solved in streamline grid while in finite difference simulation, they are in finite difference grid. In other words, streamline simulation is faster than finite difference simulation because of changing 3D saturation equations into 1D equation along streamline. In streamline simulation 3D spaces are changed into 1D space. In streamline simulation we can run simulators only for a short period (e.g. One day) and investigate the progress of fluids (water injection) during simulation (ten years). This is a special character for streamline simulation that is illustrated with time of flight. Streamline simulators show the relationship between injection wells and producer wells. This character is specified for streamline simulation and by using it we can determine optimization for the locations of the injector wells and decrease the rate of water injection and water cut in our reservoir.

### Nomenclature

D = depth from datum, L

$f_j$  = fractional flow of phase j, dimensionless

$\vec{G}_j$  = gravity fractional flow term of phase j, L/T

g = gravitational acceleration constant, L/T<sup>2</sup>

$\vec{k}$  = absolute permeability tensor, L<sup>2</sup>

$k_{ij}$  = relative permeability of phase j, dimensionless

n = number of pressure time steps

$n_p$  = number of phases

P = Pressure M/T<sup>2</sup>L

$S_j$  = saturation of phase j, dimensionless

$S_j^c$  = intermediate saturation profile after convective step

$s$  = spatial distance coordinate along a streamline, L

$t$  = time, T

$\vec{u}_t$  = total Darcy velocity, L/T

$z$  = vertical coordinate direction, L

$\Delta t$  = time step size between pressure solves, T

$\Phi$  = porosity, dimensionless

$\rho_j$  = density of phase  $j$ , M/L<sup>3</sup>

$\zeta$  = local streamline coordinate, L

$\lambda_g$  = total gravity mobility, L/T

$\lambda_t$  = total mobility, L<sup>3</sup>T/M

$\tau$  = time of flight, T

$\mu_j$  = viscosity of phase  $j$ , M/TL

## 8. Conclusion

There is no doubt that streamline is a powerful tool for analysing waterflooding performance in heterogeneous oil reservoirs, but this study demonstrates its efficiency and the practical implication of applying simulation only to a limited domain. Our comparison shows that although streamline and conventional finite difference simulations produced results with similar trends of reservoir pressure and oil production, the streamline method has superior visualization of fluid flow paths, injector–producer connectivity, allocation factors, and time-of-flight distributions enabling better-informed decision making regarding tight reservoir management. In practice, optimizing injection rates based on streamline connectivity led to a significantly decreased field water cut (from 0.62 to 0.49) and cumulative water production (reducing these volumes of produced water from 83.03 m<sup>3</sup> to 58.31 m<sup>3</sup>), while sustaining satisfactory oil recovery performance; this illustrates the potential of streamline simulation in increasing sweep efficiency and dampening excessive water production; respectively. These results demonstrate that, in addition to improving computational efficiency through a transformation of three-dimensional transport problems into one-dimensional flow calculations, streamline-based reservoir simulation offers diagnostic information that may not be easily derived using traditional finite difference methods. The study has important practical implications and highlights the significance of incorporating streamlined simulation into surveillance, waterflood optimization, and field development planning; as it provides the ability to enhance injection strategies while maximizing hydrocarbon recovery and minimize water handling costs in heterogeneous reservoirs. However, this study focused on one offshore reservoir model in certain geological and operational contexts. Hence, the future works should focus on further exploration of streamline simulator under more complex reservoir environments such as naturally fractured formation, multiphase compositional flow, enhanced oil recovery processes, uncertainty analysis and optimization assisted by machine learning to enhance prediction results and support an intelligent reservoir management in the production dynamic environment.

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