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Mathematical Model for Improving Wear Resistance and Reducing Noise in Bevel Gear Transmissions

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Abstract: This paper presents a mathematical model aimed at evaluating the geometric accuracy of tooth surfaces and reducing wear and noise in bevel and hypoid gear transmissions used in automobile drive axles. The study considers the spatial kinematic and geometric characteristics of the grinding process based on the formate–tilt principle, in which the generating motions between the cutting tool and the workpiece are modeled using coordinate systems. Based on the parametric equations of the cutter head surface, the spatial theoretical equations of the gear tooth surface are derived, and the geometric deviations between mating tooth surfaces are evaluated using a discretization approach. The proposed model can be applied to analyze the meshing process, assess load distribution, and substantiate technological solutions aimed at reducing noise levels in bevel and hypoid gear transmissions.

Keywords: bevel gear; hypoid gear; tooth surface; grinding; mathematical model; meshing; noise.

Introduction. In the manufacturing of hypoid gear transmissions used in automobile drive axles, grinding technologies based on the formate–tilt principle are widely applied in practice to achieve high geometric accuracy. These technologies improve the accuracy of tooth surface formation and ensure high efficiency of the machining process [1, 2].

Within this approach, hypoid gear transmissions employed in automotive drivetrains are commonly manufactured using grinding processes in which the large gear is generated by the formate method, whereas the small gear is produced using a generating method that accounts for the tilt angle of the cutting tool. During the grinding process, the rotation of the cutting tool about its own axis results in the formation of a virtual auxiliary gear. The geometric shape of the machined tooth surface is generated through the generating rotational motion of this auxiliary gear.

Research Methods. To develop a mathematical model of the grinding process, several coordinate systems were introduced. Specifically, the coordinate system $S_m(X_m, Y_m, Z_m)$ is associated with the machine tool and represents its overall motion, $S_c(X_c, Y_c, Z_c)$ is linked to the rotating table, and $S_1(X_1, Y_1, Z_1)$ moves together with the machined gear. In addition, auxiliary coordinate systems Si, Sj, Ss, and Sh were employed to accurately describe spatial transformations.

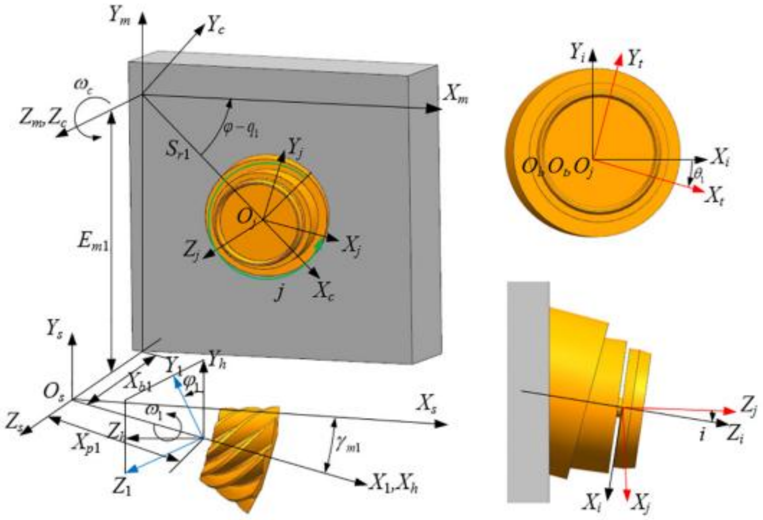

Figure 1. Spatial mathematical model of the grinding process with tool tilt angle.

The model parameters include the tool tilt angle, the main rotation angle, the radial distance, the offset distance from the axis, and the structural angles of the machine tool. The spatial mathematical model of the grinding process is shown in Figure 1.

In the spatial model shown in Figure 1, i denotes the tilt angle of the cutter head, j is the main rotation angle, S_{r1} is the radial distance, q_1 is the rotation angle of the main table, E_{m1} represents the offset distance of the driving bevel gear, X_{b1} is the sliding base, X_{p1} is the distance from the machine center to the cutting point, and $\gamma m1$ is the root angle of the machine tool.

In the initial machining position, the coordinate system $S_1(X_1, Y_1, Z_1)$ coincides with the coordinate system $S_h(X_h, Y_h, Z_h)$. Under this condition, the angle between the X_c and X_m axes is equal to q_1 [3, 4]. When the rotating table turns about its axis, this angle changes to $\varphi - q_1$, while the workpiece simultaneously rotates about its own axis by an angle φ_1. According to the generating motion relationship, $\varphi_1 = R_{b\varphi}$, where R_b denotes the rotation ratio.

Discrete Calculation Method for the Tooth Surface. Since the tooth surfaces of hypoid gears have a complex spatial geometry, the determination of an accurate geometric model requires calculating the coordinates of grid points on the tooth surface [2, 5]. Therefore, a discretization method is applied, in which the tooth surface is divided into a grid. This process is illustrated in Figure 2.

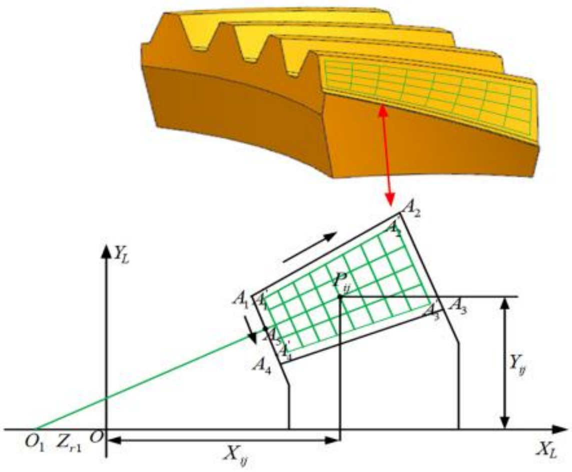

Figure 2. Discretization of the tooth surface into grid points.

The two-dimensional coordinates of each grid point are calculated based on the geometric parameters of the gear. Subsequently, a system of equations is formulated using the correspondence between the points on the spatial tooth surface and the points on a section parallel to the tooth axis. Through this procedure, the three-dimensional (3D) coordinates of the tooth surface are determined.

Results and Discussion. In accordance with the geometric structure of the cutter head, the spatial position of its surface is determined by defining the parametric equation of the cutter head surface together with the corresponding unit normal vector. The parametric equation of the cutter head surface expressed in the coordinate system $S_t(X_t, Y_t, Z_t)$ is given by equation (1), while the corresponding unit normal vector is defined by equation (2) in the following form:

$$r_t\,(u_1, \theta_1) = \begin{bmatrix} (r_p + u_1 \sin\alpha_1)\cos\theta_1 \\ (r_p + u_1 \sin\alpha_1)\sin\theta_1 \\ -u_1 \cos\alpha_1 \\ 1 \end{bmatrix} \quad (1)$$

$$n_t = [\, -\cos\alpha_1 \cos\theta_1 \text{-} \cos\alpha_1 \sin\theta_1 \sin\alpha_1 \,] \quad (2)$$

Here, (u_1, θ_1) denote the parametric coordinates of the cutter head surface, α_1 is the profile angle of the cutter head, and r_p represents the radius of the cutter head tip (point).

The equation of the gear tooth surface is obtained through successive coordinate transformations from the tool surface. Based on the relationships between the coordinate systems, the spatial equation of the tooth surface and its corresponding unit normal vector are determined using the following spatial transformation relationship:

$$r_1(u_1, \theta_1, \varphi_1) = M_{1h} M_{hs} M_{sm} M_{mc} M_{cj} M_{ji} M_{it} r_t\,(u_1, \theta_1) \quad (3)$$

$$n_1(u_1, \theta_1, \varphi_1) = L_{1h} L_{hs} L_{sm} L_{mc} L_{cj} L_{ji} L_{it} n_t\,(u_1, \theta_1) \quad (4)$$

Here, $M_{1h}, M_{hs}, M_{sm}, M_{mc}, M_{cj}, M_{ji}$ and M_{it} are the transformation matrices between the corresponding coordinate systems, while $L_{1h}, L_{hs}, L_{sm}, L_{mc}, L_{cj}, L_{ji}$ and L_{it} are the corresponding 3×3 submatrices obtained by removing the elements of the last row and the last column of each transformation matrix.

During the generating motion between the tool and the workpiece, the meshing condition between the teeth must be satisfied. This condition ensures that there is no relative displacement between the tooth surfaces along the normal direction, and the corresponding kinematic relationship can be expressed as follows:

$$f_1 = (u_1, \theta_1, \varphi_1) = n_1 \cdot v^{(p1)} = \frac{\dfrac{\partial r_1}{\partial \theta_1} \times \dfrac{\partial r_1}{\partial u_2}}{\left| \dfrac{\partial r_1}{\partial \theta_1} \times \dfrac{\partial r_1}{\partial u_2} \right|} \cdot \frac{\partial r_1}{\partial \varphi_1} \cdot \varphi_1 = 0 \quad (5)$$

Here, v^{p1} denotes the relative velocity of motion between the generating large gear and the machined workpiece.

Equation (5) represents the kinematic constraint between the tool and the workpiece, ensuring that during the formation of the tooth surface there is no relative displacement between the mating surfaces in the normal direction, that is, the surfaces remain in contact throughout the generating process.

The mathematical model of grinding that accounts for the cutter head tilt angle, shown in Figure 1, constitutes a unified model that is primarily applied to the grinding of the small gear. However, due to its structural formulation, this model can also be applied to the grinding of the large gear. In this case, only the grinding parameters corresponding to the gear need to be substituted into equations (3) and (4). As a result, the spatial equation of the gear tooth surface and its corresponding unit normal vector are determined. These are denoted as $r_2(u_2, \theta_2)$ and $n_2(u_2, \theta_2)$, which represent the vector equation of the gear tooth surface and the normal direction vector, respectively.

In Figure 2, X_L denotes the direction of the tooth axis, and Y_L denotes the direction perpendicular to the tooth axis. Point O_1 represents the apex point of tooth rotation, O is the constructional center, and Zr_1 is the distance from O_1 to O. Points A_1, A_2, A_3, and A_4 define the boundary points of the tooth surface, while A'_1, A'_2, A'_3, and A'_4 correspond to the boundary points after truncation of the tooth surface.

If the coordinates of an arbitrary grid point P_{ij} in the axial section of the tooth surface are given in the form (X_{ij}, Y_{ij}), the corresponding point on the spatial tooth surface can be expressed as r_P $(x(u_P, \theta_P, \varphi_P), y(u_P, \theta_P, \varphi_P), z(u_P, \theta_P, \varphi_P))$

The relationship between the three-dimensional coordinate points on the spatial tooth surface and the points in the axial section is described by equation (6):

$$\begin{cases} x(u_P,\theta_P,\varphi_P) = X_{ij} \\ y^2(u_P,\theta_P,\varphi_P) + z^2(u_P,\theta_P,\varphi_P) = Y_{ij}^2 \\ f_1(u_P,\theta_P,\varphi_P) = 0 \end{cases} \quad (6)$$

This equation represents a nonlinear system, which is solved using the iterative Newton–Raphson algorithm. The computational procedure is implemented in the VC programming environment. As a result, by solving equation (6), the spatial three-dimensional coordinates of any grid point on the tooth surface are determined.

Conclusion. As a result of the study, a spatial mathematical model of the tooth surface was developed for a grinding process based on the formate–tilt principle. The theoretical shape of the tooth surface was determined using the parametric equations of the cutter head surface and coordinate transformations, and the geometric deviations between mating tooth surfaces were quantitatively evaluated using a discretization approach. The obtained results provide a basis for analyzing the meshing process and for substantiating technological solutions aimed at reducing wear and noise in bevel and hypoid gear transmissions.

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