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Methods of Synthesis of Regulators for Nonlinear Dynamic Systems

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Abstract: Methods and algorithms for the synthesis of regulators for nonlinear dynamic systems are given, based on various approaches to the formation of a control device. We considered synthesis methods based on phase space, Lyapunov function method, vibration control, feedback linearization method, velocity gradient method, lag feedback method, adaptive, robust, intelligent approaches, fuzzy logic, systems with a variable structure, etc. All the methods of synthesis of nonlinear regulators discussed above give a solution to the problem only for rather narrow classes of objects. Therefore, the problem of building regulators for nonlinear systems in many cases has not yet been solved.

Keywords: nonlinear dynamic object, regulator, regulator synthesis, control.

The theory of linear systems is the most developed branch of the theory of automatic control due to the fact that the use of the superposition principle allows the use of a convenient apparatus of transfer functions and state space. Meanwhile, real control objects can detect significant nonlinear properties, which must be taken into account when designing control systems thereof.

To date, a large number of methods have been created for the synthesis of nonlinear automatic control systems. Taking into account the influence of nonlinearities in any automatic control system faces great difficulties, since you have to deal with the solution of non-linear differential equations of higher orders. The choice of a particular method depends on the setting of the study problem, the type of nonlinearity and the order of the differential equation describing the system.

If the control system is described by a differential equation of the first, second or third order, then methods based on the study of processes in phase space are used to analyze and synthesize nonlinear systems. The study of processes in phase space refers to the exact methods of studying nonlinear systems, since it allows you to obtain exactly the necessary and sufficient stability conditions. This method is distinguished by clarity and the ability to obtain a complete idea of the nature of possible states of the system. The method is based on the concept of phase space.

Powerful tools for the analysis and synthesis of nonlinear control systems are the Lyapunov function method (direct or second Lyapunov method) [1-3] and frequency methods combined by the Yakubovich-Kalman lemma [4].

The conventional perturbation control principle [5-7] assumes the effect on the nonlinear system of a preselected external signal $u(t)$ as some function of time without taking into account the values of the controlled process. This can be either a certain physical impact on the system or a change in some parameter of the managed system. The advantage of this control principle is ease of implementation, since it does not require measurements and installation of sensors, which is important when controlling ultrafast processes, for example, occurring at the molecular or atomic level, for which there is no possibility of measuring the state of the system in real time.

As noted in work [5], the possibility of a significant change in the dynamics of the system by a periodic excitation signal has been known for a long time. Back in the first half of the 20th century. The possibility of stabilizing the pendulum in an unstable state using a high-frequency signal was shown, which laid the foundation for vibration mechanics. At the same time, the effect of high frequency excitation on the behavior of non-linear systems of the general type was investigated using the Krylov-Bogolyubov averaging method. In control theory, high-frequency impacts are used in the construction of vibration and the so-called "trembling" control (dither control), as well as in the recent works of G.A. Leonov during transient stabilization. At the same time, the task of stabilizing the system relative to a given state of equilibrium or trajectory was set. In recent publications, for systems presented in the form of Lurie, it is proposed to use vibrating control with a piecemeal constant input stochastic ("trembling") signal.

For view systems

$$\dot{x} = f(x) + Bu, x \in R^n, u \in R^m, \quad (1)$$

where $m = n$ and $\det B \neq 0$, work [8,9] proposes the construction of a combined control called "openplus- closed-loop" (OPCL). In this case, the law of management is sought as

$$u(t) = B^{-1}\big(\dot{x}_*(t) - f(\dot{x}_*(t)) - K(\dot{x} - \dot{x}_*(t))\big), \quad (2)$$

where K - square matrix of gains.

A number of methods have been developed to construct the control of nonlinear objects with incomplete measurement. In particular, the feedback linearization method [5] allows, using the methods of linear system theory, to ensure the desired dynamics of a closed system. Consider the idea of the method for systems affinity for control

$$\dot{x} = f(x) + g(x)u, x \in R^n, u \in R^m. \quad (3)$$

The system (3) is linearizable by feedback in the region $\Omega \in R^n$ if there is a smooth reversible coordinate replacement $z = \Phi(x), x \in \Omega$, and a smooth feedback transformation

$$u = \alpha(x) + \beta(x)v, x \in \Omega \quad (4)$$

where $v \in R^m$ - new control such that the closed system is linear, i.e. its closed coordinate equation is

$$\dot{z} = Az + Bv \quad (5)$$

for some constant matrices A, B. A significant drawback of this method is that such an approach ignores the system's own dynamics. Arbitrary desired dynamics are achieved at the cost of high control power required under significant initial conditions and when tracking rapidly changing program motion.

A number of methods for constructing a nonlinear control are based on changing the value of some objective function $Q(x(t), t)$ [5]. For example, the value $Q(x(t), t)$ may be the distance between the state of the system at a given time $x(t)$ and the current point $x_*(t)$ on a given path $Q(x(t), t) = \|x - x_*(t)\|^2$, where $\| x \|$ is the Euclidean norm of vector x.

Also, a distance $Q(x) = \| h(x) \|^2$ from the current position of the system $x(t)$ to a given target surface $h(t) = 0$ can be used as an objective function. For continuous time systems, the value of $Q(x)$ does not directly depend (at the same time) on the control signal and, therefore, instead of $Q(x)$, one can consider its derivative $\dot{Q}(x) = (\partial Q / \partial x)F(x, u)$, i.e., reduce the rate of change of the objective function over time.

Based on this, the methods of the velocity gradient (VG method) [10], which assume a change in control and in the direction of the anti-gradient along and the speed $Q(x)$ of the original objective function. In particular, algorithms in the so-called finite form have the form

$$u = -\psi\big(\nabla_u \dot{Q}(x, u)\big) \quad (6)$$

where $\psi(z) -$ is some vector function whose value is directed at an acute angle to its argument z. For affine control objects (3), the algorithm (6) takes the form

$$u = -\psi(g(x)^T \nabla Q(x)) \quad (7)$$

In this case, Lyapunov V functions are used as an objective function, decreasing along the trajectories of the closed system.

A relatively new task is to develop methods of synthesis of nonlinear systems by ensuring the passivity of this nonlinear system using feedback or by selecting a special output function [5,11,12]. The idea of the approach formed to date is that the task of stabilizing a nonlinear controlled system is solved using the following two-stage procedure. At the first stage, the task of passigating a nonlinear system is set, which consists in ensuring the passivity of the original system. For an affine control system (3), this means the existence of a function $V(x)$ and a feedback (4) such that

$$\dot{V}(x) = \frac{\partial V}{\partial x}(f + g\alpha + g\beta v) \leq yv. \quad (8)$$

It can be said that (8) is performed if the yield y is taken as $y = L_g V\beta$. This is a velocity gradient algorithm at $x \neq 0$ and $\dot{V}\big|_{y=0} < 0$. The latter condition means that the so-called zero-dynamics of the system, i.e. the motion on the manifold $y = 0$, is asymptotically stable (this property is called a minimum phase) [12].

At the second stage, when additional conditions of the observability type are met, the task of stabilizing the passive system obtained as a result of the first stage is solved. The advantages of this method are the division of the initial complex problem into two simpler and more versatile first stage, since the passivation of the system can also be an intermediate goal in solving problems other than the stabilization problem. Another advantage of the passivation method is that it does not require explicit calculation of the Lyapunov function to synthesize the system and investigate it.

As noted in work [5], in recent years, interest in the time-delayed feedback method proposed by K. Piragas has increased [13]. He considered the task of stabilizing the unstable τ-periodic orbit of a nonlinear system

$$\dot{x}(t) = F(x, u) \quad (9)$$

using a simple feedback law:

$$u(t) = K(x(t) - x(t - \tau)) \quad (10)$$

where $K -$ transmission ratio, τ - delay time. If τ is equal to the period of the existing periodic solution $\bar{x}(t)$ of equation (9) at $u = 0$ and the solution $x(t)$ of the closed system equation (9), (10) begins in orbit $\Gamma = \{\bar{x}(t)\}$, then it remains in Γ for all $t \geq 0$. Decision $x(t)$ may converge to Γ even if $x(0)\bar{\epsilon}\Gamma$. A disadvantage of the control law (10) is its sensitivity to parameter selection, especially τ. Despite the simple form of the algorithm (10), analytical research by a closed system is a complex and not yet fully solved problem.

Currently, technological progress leads to a reduction in the time taken to create modern systems, which creates significant difficulties in creating mathematical models of processes and control objects. Therefore, many modern self-propelled guns are created in conditions of a priori uncertainty. This means that some of the characteristics of the control object can be previously unknown or change during its operation. To solve the problem of controlling dynamic systems in conditions of uncertainty, such basic approaches as adaptive (self-organizing) [10,14-16], robust [17,18], intelligent (based on neural-like networks) [19, 20], invariant, principles of fuzzy logic (fuzzy controllers) [21], principles of systems with a variable structure have been developed. However, in the non-linear case, this problem does not currently have a complete solution at the level of modern requirements.

Adaptive systems are designed to operate in the presence of recoverable uncertainties in the system [16], i.e. those that are a priori unknown, but can be estimated or calculated during operation from measurement data. Traditional adaptive control techniques assume that the order of the system is known and does not change during its operation. Another limitation of the use of adaptive systems is the need for the quasi-stationary nature of undefined objects, i.e. the slowness of changing their parameters.

The robust approach is mainly used for systems with parametric perturbations of small magnitude.

The works [6,22,23] examined systems with spontaneously changing structure and order. In particular, in [6] some features of systems with hopping state variables are noted.

In recent years, H_∞-optimal synthesis [24] has been intensively developed, which allows to obtain a solution in the case of non-linear systems with uncertainties. The main tool of this approach is a set of manifolds introduced in the state space of a system, on which given relations between variables are performed in steady mode. However, here it is necessary to solve non-linear inequalities (such as the Hamilton-Jacobi equation) in partial derivatives, which greatly complicates the practical application of this method.

[25-27] proposes an approach that implements control over the derivatives of the observed variable and uses the representation of the system variables by the Taylor finite series. Since the Taylor series of a differentiable function is based on its derivatives, one of the advantages of this concept is that it leads to such models whose state variables coincide or are directly related to the time derivatives of the observed variables [6,28]. Based on the information received from the measuring elements, the self-organizing regulator determines both its parameters and its structure.

A widespread approach to the design of nonlinear regulators is based on the representation of the regulator as a neural network structure in which nonlinear activation functions are configured. At the same time:

1. An arbitrary nonlinear control law is described by a hypersurface in n-dimensional space.

2. The description of the control hypersurface is formed as the sum of the nonlinear control laws for each measurement separately.

3. Adjustment of nonlinear fuzzy regulator is equivalent to optimization of nonlinear activation function parameters.

When optimizing the linear control law, the parameters are represented by a set of constants, where the one constant corresponds to one control channel.

When optimizing the nonlinear control law, a whole set of constants $P = \{p_1, p_2, \dots, p_k\}$ will correspond to one channel. The objective function can be selected as:

$$F(p_1, p_2, \dots, p_k) = \sum_{i=1}^{N} |y_i^* - y_i|,$$

where y and y^*-real and desired value of object output, i - moment of time.

The objective function is (4) multimodal, which requires the use of global optimization techniques such as a genetic algorithm [29].

The following algorithm can be used to simplify the global optimization problem in some cases [30,31]:

1. A linear PID is synthesized - a regulator (linear neural network), the parameters of which k_p, k_i, k_d, will play the role of denormalization coefficients.

2. Non-linear functional dependencies are configured that describe a fuzzy control law (activation functions) for each of the input variables.

This algorithm was used to control the active suspension of the vehicle [32], nonlinear objects such as Gammerstein and Wiener [33], nonlinear oscillator [30]. The modeling examples given in these papers show that the use of nonlinear regulators can provide a significant reduction in transient time and a reduction in overregulation, which is not achievable under a linear control law.

Thus, the proposed approach can be useful in modernizing control systems of a wide class of dynamic objects where linear PID regulators are used.

In control theory, variable-structure regulators are known in which line blocks are switched [34-36]. These regulators are built on the basis of phase portraits of blocks, so they usually commute secondorder line blocks. This causes the narrow possibilities of such regulators, especially for the synthesis of control systems for non-linear objects.

The synthesis method proposed in $[34-36]$ is characterized by the fact that nonlinear control laws are committed. Therefore, the corresponding regulators are called nonlinear variable structure regulators (NVSR). The synthesis of commutable nonlinear control laws is based on the Lyapunov function, which significantly expands the control capabilities of nonlinear objects. The NVSR algorithm is quite complex, but it is easily implemented on industrial microcontrollers. Thus, the method proposed in [33,34] makes it possible to use the wide possibilities of modern computer technology.

This algorithm of operation of the nonlinear variable structure regulator provides effective control of nonlinear objects, and can be extended to the case of objects with several controls.

All the methods of synthesis of nonlinear regulators discussed above give a solution to the problem only for rather narrow classes of objects. Therefore, the problem of building regulators for nonlinear systems in many cases has not yet been solved.

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