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# Dynamic Calculation of a Multi-Storey Building under Longitudinal-Shear Vibrations

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**Abstract:** Within the framework of the bimoment theory of thick plates, a spatial dynamic continuum model of multi-storey buildings under longitudinal seismic impact is proposed. Formulas are proposed for determining the reduced moduli of elasticity and density. The natural frequency, displacement and stress values of a multi-story building were calculated.

**Keywords:** Seismic load, multi-story building, displacements, stresses, force, bimoment, bimoment theory, plate model, longitudinal vibrations, equations of motion, mesh method.

**Introduction.** At the present stage of development of the theory of seismic resistance, one of the urgent tasks of the science of dynamics of structures is to improve methods for calculating multi-story and high-rise buildings, since the Republic of Uzbekistan is located in a seismically active zone of the globe. Many articles and monographs by researchers from foreign countries and our republic have been published on the calculations of multi-story and high-rise buildings.

Article [1] presents the formulation of problems for calculating a structure under seismic impacts, and outlines methods for solving them using the finite element method. Methods for determining the parameters of seismic impact based on the application of probability theory and the theory of random processes are proposed in [2]. Based on the results of the analysis of a number of accelerograms, the parameters of the seismic impact were determined.

Articles [3,4] present the basic principles of modeling multi-storey buildings with seismic isolation devices. Methods for conducting experimental studies on a laboratory vibration stand under dynamic (seismic) influences and modeling results are proposed.

Articles [5-6] are devoted to the dynamic calculation of box-shaped building structures for seismic resistance, taking into account the spatial operation of box-shaped elements under the influence of dynamic influence. A mathematical model and a numerical-analytical method for solving the problem of dynamics using the finite difference method and expanding the solution according to the modes of natural vibrations in the spatial formulation of elements of box-shaped structures under kinematic influence have been developed.

Statement and solution of the problem.

When constructing a plate model of a building, two main problems arise. The first problem is related to the determination of the reduced moduli of elasticity and density of multi-story buildings as a continuous three-dimensional body. The second problem is determining the boundary conditions on the side faces and at the top end of the building.

The problem of longitudinal vibrations of a multi-story building is a symmetric problem of the bimoment theory of plate structures, developed in [7-9]. The system of equations for longitudinal vibrations of multi-story and high-rise buildings is described by nine unknown kinematic functions, determined by the formulas:

$$\bar{\psi}_k = \frac{1}{2h}\int_{-h}^{h} u_k dz, \quad \bar{\beta}_k = \frac{1}{2h^3}\int_{-h}^{h} u_k z^2 dz, \quad (k=1,2),$$
$$\bar{r} = \frac{1}{2h^2}\int_{-h}^{h} u_3 z dz, \quad \bar{\gamma} = \frac{1}{2h^4}\int_{-h}^{h} u_3 z^3 dz. \qquad (1)$$

To describe the problem of longitudinal vibrations of a building, longitudinal and tangential forces are introduced in a continuous plate model of a multi-story building $N_{11}, N_{12}, N_{22}$ от stress $\sigma_{11}, \sigma_{12}, \sigma_{22}$, which are defined by the expressions:

$$N_{11} = E_{11}H\frac{\partial \bar{\psi}_1}{\partial x_1} + E_{12}H\frac{\partial \bar{\psi}_2}{\partial x_2} + 2E_{13}\bar{W},$$
$$N_{22} = E_{12}H\frac{\partial \bar{\psi}_1}{\partial x_1} + E_{22}H\frac{\partial \bar{\psi}_2}{\partial x_2} + 2E_{23}\bar{W}, \; N_{12} = N_{21} = G_{12}\left(H\frac{\partial \bar{\psi}_2}{\partial x_1} + H\frac{\partial \bar{\psi}_2}{\partial x_1}\right). \qquad (2)$$

Similarly, expressions for the bimoments generated by longitudinal vibrations of the plate model of the building are constructed, $T_{11}, T_{22}, T_{12}$ from stress $\sigma_{11}, \sigma_{12}, \sigma_{22}$, they are defined as:

$$T_{11} = H\left(E_{11}\frac{\partial \bar{\beta}_1}{\partial x_1} + E_{12}\frac{\partial \bar{\beta}_2}{\partial x_2} + E_{13}\frac{2\bar{W} - 4\bar{r}}{H}\right),$$
$$T_{12} = T_{21} = HG_{12}\left(\frac{\partial \bar{\beta}_2}{\partial x_1} + \frac{\partial \bar{\beta}_1}{\partial x_2}\right), \quad T_{22} = H\left(E_{12}\frac{\partial \bar{\beta}_1}{\partial x_1} + E_{22}\frac{\partial \bar{\beta}_2}{\partial x_2} + E_{23}\frac{2\bar{W} - 4\bar{r}}{H}\right). \qquad (3)$$

Expressions for the intensities of transverse bimoments generated during longitudinal vibrations of a plate model of a building are introduced, $\bar{p}_{13}, \bar{p}_{23}$ and $\bar{\tau}_{13}, \bar{\tau}_{23}$ from tangential stresses $\sigma_{13}, \sigma_{23}$, determined by the formulas:

$$\bar{p}_{k3} = G_{k3}\left(\frac{\partial \bar{r}}{\partial x_k} + \frac{2(\bar{u}_k - \bar{\psi}_k)}{H}\right), \quad \bar{\tau}_{k3} = G_{k3}\left(\frac{\partial \bar{\gamma}}{\partial x_k} + \frac{2(\bar{u}_k - 3\bar{\beta}_k)}{H}\right), \quad (k=1,2). \qquad (4)$$

The concept of bimoment intensities is also introduced $\bar{p}_{33}$ и $\bar{\tau}_{33}$ from normal voltage $\sigma_{33}$ according to the following formulas:

$$\bar{p}_{33} = E_{31}\frac{\partial \bar{\psi}_1}{\partial x_1} + E_{32}\frac{\partial \bar{\psi}_2}{\partial x_2} + E_{33}\frac{2\bar{W}}{H}, \; \bar{\tau}_{33} = E_{31}\frac{\partial \bar{\beta}_1}{\partial x_1} + E_{32}\frac{\partial \bar{\beta}_2}{\partial x_2} + E_{33}\frac{2\bar{W} - 4\bar{r}}{H}. \qquad (5)$$

Now, let us present a system of equations for longitudinal vibrations of a multi-story building. The system of differential equations of seismic vibrations of a multi-story building relative to longitudinal and tangential forces (2) is constructed in the form:

$$\frac{\partial N_{11}}{\partial x_1} + \frac{\partial N_{12}}{\partial x_2} = \rho H \ddot{\bar{\psi}}_1, \quad \frac{\partial N_{21}}{\partial x_1} + \frac{\partial N_{22}}{\partial x_2} = \rho H \ddot{\bar{\psi}}_2. \qquad (6)$$

The system of differential equations of seismic vibrations of a multi-story building relative to longitudinal and tangential bimoments (3)-(5) generated during longitudinal vibrations of the building is constructed in the form

$$\frac{\partial T_{11}}{\partial x_1}+\frac{\partial T_{12}}{\partial x_2}-4\overline{p}_{13}=\rho H\ddot{\overline{\beta}}_1,\quad \frac{\partial T_{12}}{\partial x_1}+\frac{\partial T_{22}}{\partial x_2}-4\overline{p}_{23}=\rho H\ddot{\overline{\beta}}_2. \qquad (7)$$

$$\frac{\partial \overline{p}_{13}}{\partial x_1}+\frac{\partial \overline{p}_{23}}{\partial x_2}-\frac{2\overline{p}_{33}}{H}=\rho\ddot{\overline{r}}\,, \qquad (8)$$

$$\frac{\partial \overline{\tau}_{13}}{\partial x_1}+\frac{\partial \overline{\tau}_{23}}{\partial x_2}-\frac{6\overline{\tau}_{33}}{H}=\rho\ddot{\overline{\gamma}}\,. \qquad (9)$$

It should be noted that the systems of four equations (5) - (9) contain new six unknown functions $\overline{u}_1,\ \overline{u}_2,\ \overline{\beta}_1\ \ \overline{\beta}_2,\ \overline{r},\ \overline{\gamma}$.

Thus, the systems of six equations (5)-(9) contain nine unknown functions $\overline{u}_1,\ \overline{u}_2,\ \overline{\beta}_1\ \ \overline{\beta}_2,\ \overline{r},\ \overline{\gamma},\ \overline{\psi}_1,\ \overline{\psi}_2,\ \overline{W}$. Based on the application of this expansion method, three approximate equations were constructed to determine generalized displacements $\overline{u}_1,\ \overline{u}_2,\ \overline{W}$, which will be rewritten as follows:

$$\overline{u}_k=\frac{1}{4}\left(21\overline{\beta}_k-3\overline{\psi}_k\right)-\frac{1}{20}H\frac{\partial\overline{W}}{\partial x_k}\quad (k=1,2), \qquad (10)$$

$$\overline{W}=\frac{1}{2}\left(21\overline{\gamma}-7\overline{r}\right)-\frac{1}{30}H\left(\frac{E_{31}}{E_{33}}\frac{\partial\overline{u}_1}{\partial x_1}+\frac{E_{32}}{E_{33}}\frac{\partial\overline{u}_2}{\partial x_2}\right).. \qquad (11)$$

To represent the boundary conditions of the problem of longitudinal vibrations for multi-story buildings (10) - (11), we introduce the intensities of bimoments $\overline{\sigma}_{11},\ \overline{\sigma}_{22},\ \overline{\sigma}_{12},\ \overline{\sigma}^*_{11},\ \overline{\sigma}^*_{22}$, which are determined by the formulas obtained in [7-9]. Bimoments Now we write down the boundary conditions on the free side and top faces of a multi-story building. On the side faces, the conditions for forces, moments and bimoments to be zero are set:

$$N_{11}=0,\ N_{12}=0,\ T_{11}=0,\ T_{12}=0,\ \overline{p}_{13}=0,\ \overline{\tau}_{13}=0,\ \overline{\sigma}_{11}=0;\ \overline{\sigma}_{12}=0\ \overline{\sigma}^*_{13}=0. \qquad (12)$$

On the free upper face, the conditions for forces and bimoments to be zero are also specified:

$$N_{12}=0,\ N_{22}=0,\ T_{12}=0,\ T_{22}=0,\ \overline{p}_{23}=0,\ \overline{\tau}_{23}=0,\ \overline{\sigma}_{12}=0;\ \overline{\sigma}_{22}=0\ \overline{\sigma}^*_{23}=0. \qquad (13)$$

Initial conditions, i.e. values of the sought functions at $t_0=0$ in longitudinal vibrations of the building are considered zero:

$$\begin{aligned}&\overline{\psi}_1=0,\ \overline{\psi}_2=0,\ \overline{\beta}_1=0,\ \overline{\beta}_2=0,\ \overline{r}=0,\ \overline{\gamma}=0,\ \overline{u}_1=0,\ \overline{u}_2=0,\ \overline{W}=0,\\&\dot{\overline{\psi}}_1=0,\ \dot{\overline{\psi}}_2=0,\ \dot{\overline{\beta}}_1=0,\ \dot{\overline{\beta}}_2=0,\ \dot{\overline{r}}=0,\ \dot{\overline{\gamma}}=0,\ \dot{\overline{u}}_1=0,\ \dot{\overline{u}}_2=0,\ \dot{\overline{W}}=0.\end{aligned} \qquad (14)$$

The problem was solved based on the use of the numerical finite difference method, using which an algorithm and program for calculating natural frequencies, displacement and stress at characteristic points of a multi-story building were developed.

**Numerical results and their analysis**. Calculations for a multi-storey building were made under the assumption that seismic soil movement occurs along the length of the building in the form of movement of the building's base:

$$u_0(t)=\frac{A_0}{2}(1-\cos(\omega_0 t)). \qquad (15)$$

Where $A_0=\frac{2a_0}{\omega_0^2}$ – amplitude of movement of the base. Where $a_0$ - acceleration amplitude and $\omega_0=2\pi\nu_0$ – circular frequency of the soil base.

Let us present the results of calculations of natural frequencies, displacements and stresses for a 12-story building. Using the resonance method, the values of the natural frequency of a twelve-story building are calculated depending on two values of the building width $H = 11\text{m}$, $H = 13m$ и $H = 15m$.

Figures 1 and 2 show graphs of changes in normal displacement $\psi_1$ in the middle of the top floor of a twelve-story building and the maximum normal stress $\sigma_{22}$ in the middle of the lower part of the first floor. It is clear from the graphs that when the frequency of external influence approaches the value of the natural frequency of the building $\nu_0 = p_1 = 5.02\text{ Hz}$, then there is an infinite increase in the observed values.

$\psi_1, m$

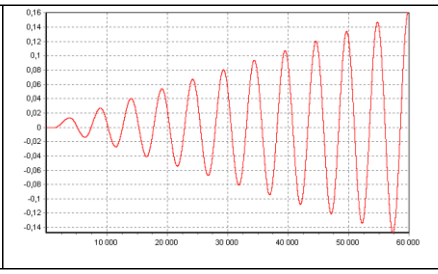


Figure 1. Graph of increasing displacement over time

σ22, MPa

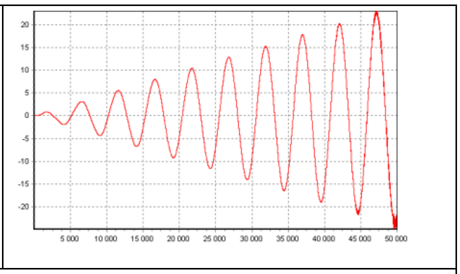


Figure 2. Graph of voltage increase over time

Based on the resonance method, the first three values of natural frequencies and periods were found for the following values of building width $H = 11\text{m}$, $H = 13\text{m}$, и $H = 15\text{m}$.

Table 1 shows three values of natural frequencies $p_1$, $p_2$, $p_3$ and periods of natural oscillations $T_1$, $T_2$, $T_3$ nine-, twelve-, sixteen-story buildings. For a nine-story and a twelve-story building, the natural frequency is determined for three options for building widths *H*=11m *H*=13m, *H*=15m. For a sixteen-story building, the frequency of natural vibrations is determined for three options for the building width *H*=13m *H*=15m,*H*=18m.

Table 1 First three natural frequencies $p_1$, $p_2$, $p_3$ and periods of natural oscillations $T_1$, $T_2$, $T_3$ nine-, twelve-, sixteen-story buildings, depending on three building widths

| № | Number of floors | *H*, m | $p_1, Hz$ | $p_2, Hz$ | $p_3, Hz$ | $T_1, sek$ | $T_2, sek$ | $T_3, sek$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 9 | 11 | 7,70 | 20,61 | 33,91 | 0,1298 | 0,0485 | 0,0295 |
| | | 13 | 7,63 | 20,31 | 33,25 | 0,1311 | 0,0492 | 0,0301 |
| | | 15 | 7,58 | 20,12 | 32,82 | 0,1319 | 0,0497 | 0,0304 |
| 2 | 12 | 11 | 5,02 | 15,31 | 28,16 | 0,1992 | 0,0653 | 0,0355 |
| | | 13 | 4,97 | 15,22 | 27,81 | 0,2012 | 0,0657 | 0,0359 |
| | | 15 | 4,93 | 15,15 | 27,42 | 0,2028 | 0,0660 | 0,0365 |
| 3 | 16 | 13 | 3,52 | 11,51 | 23,02 | 0,2841 | 0,0868 | 0,0434 |
| | | 15 | 3,44 | 11,41 | 22,83 | 0,2907 | 0,0876 | 0,0438 |
| | | 18 | 3,36 | 11,31 | 22,44 | 0,2976 | 0,0884 | 0,0446 |

In conclusion, we note that the proposed plate model of a multi-story building correctly reflects the dynamic behavior and form of vibration of the building under seismic impacts with high accuracy and determines the natural frequencies and stress-strain state of the building.

When implementing the numerical method for solving the problem, the steps in spatial coordinates are given: $\frac{c\Delta t}{\Delta x_1} = \frac{c\Delta t}{\Delta x_2} = \frac{1}{4}$.

**Conclusions and conclusions.** A spatial continuum plate model of longitudinal vibrations of multi-story buildings is proposed, developed within the framework of the bimoment theory of thick plate structures. Formulas are given for determining the elastic characteristics of a plate model of multi-story buildings, taking into account design features.

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