

Basic Methods and Tools for Solving Contact Issues

Almardonov Oybek Makhmatkulovich

Abstract: In this article, the main methods and tools for solving contact problems, which are important for the theory of elasticity, their equations and solutions are presented.

Keywords: elastic half-plane, deformation, stress, displacement components, boundary conditions, contact zone.

In solving contact problems, the stress state of the elastic half-plane under the influence of the forces placed along the cross section of the straight line is of great importance.

In the problems of the interaction of elastic bodies with inconsistent external geometry, the dimensions of the contact surface are considered to be much smaller than the dimensions of the bodies. Considering that the dimensions of the bodies are quite large, the stresses on the surface of interaction do not depend on their configurations at points far from the boundaries of the bodies, that is, one of the bodies can be considered as a semi-elastic medium. This formulation of the problem is mainly used in the "contact problems" of the theory of elasticity and makes it possible to simplify the boundary conditions and apply the mathematical methods of the theory of elasticity. Let's consider the state of stress under the influence of stresses placed on a pavement with a finite elastic half-space and a sufficiently long length (Fig. 1).

In the introduced coordinate system, the Oxy plane coincides with the boundary of the elastic half-space, and the $0z$ axis is directed inside the half-space. The width of the stress space is $a+b$ and is directed along the Oy axis. In addition, the semi-elastic space is considered to be in the state of plane deformation, that is, $\varepsilon_y = 0$.

Thus, the transverse shear of the semi-elastic space $z=0, -b \leq x \leq a$ is affected by normal and shear stresses $p(x)$ and $q(x)$, and it is required to find the stresses, $\sigma_x, \sigma_z, \sigma_{xz}$ strain tensor components, and displacement components U_x, U_z at the internal points of the transverse shear.

As mentioned above, this issue is borderline

$$\begin{cases} x < -b \\ x > a \end{cases}, \quad \overline{\sigma_z} = \overline{\tau_{xy}} = 0,$$

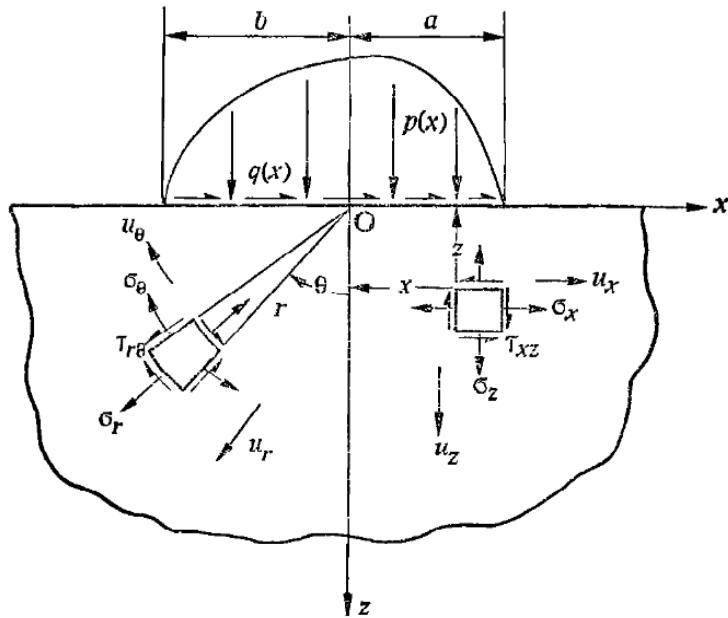
and $-b \leq x \leq a$ is biharmonic $\overline{\sigma_z} = -p(x)$, $\overline{\tau_{xz}} = -q(x)$ satisfying the conditions in the field

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} \right) = 0, \quad (1)$$

satisfying $\varphi(x, z)$

$$\sigma_z = \frac{\partial^2 \varphi}{\partial x^2}, \quad \sigma_x = \frac{\partial^2 \varphi}{\partial z^2}, \quad \tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}, \quad (2)$$

is brought to define the function.



(Fig. 1)

Furthermore, according to the problem statement, sufficiently far from the "contact zone" the stresses ($x \rightarrow \infty, z \rightarrow 0$) must satisfy the conditions $\sigma_x \rightarrow 0, \sigma_z \rightarrow 0, \tau_{xy} \rightarrow 0$.

When solving concrete problems, in most cases (stamp problems) instead of stresses at the boundary, displacements $\bar{U}_x(x)$ and $\bar{U}_x'(x)$ are given based on the geometry of the stamp, or if there is no slip under the stamp,

$$q(x) = \pm \mu p(x)$$

(μ - slip friction coefficient) if there is slip, $q(x) = 0$ conditions are taken into account.

In the case of a stamp, depending on the setting and essence of the problem, the following border conditions are mainly imposed:

In the above-mentioned general theory, we considered the state of stress under stresses applied to some areas of the half-plane. But in most cases, in contact problems, displacements are also given along with stresses at the boundary, that is, a mixed boundary problem is considered. Such a situation can be found mostly in stamp issues.

Mixed boundary conditions often take the following four forms.

1. Normal $p(x)$ and resultant $q(x)$ stresses are given at the boundary of the half-plane.
2. At the boundary of the half-plane, $\bar{u}_z(x)$ normal displacement and stress $q(x)$ or $\bar{u}_x(x)$ stress displacement and normal stress $p(x)$ are given. Such boundary conditions correspond to the absence of frictional forces between interacting surfaces and are derived from the geometrical profile of the surfaces.
3. Normal and tangential displacements $\bar{u}_z(x)$, $\bar{u}_x(x)$ are given at the boundary of the half-plane, that is, it is considered that the two surfaces do not slide relative to each other.

In this case, the frictional forces between the two surfaces will be large enough, and it is necessary to determine the normal and tensile stresses at the boundary.

4. Given the normal stress at the boundary, the connection $q(x) = \pm \mu \varphi(x)$ between the normal and test stresses is taken into account. Here, μ – is the coefficient of friction.

References:

1. Джонсон К. “Механика контактного взаимодействия”. Москва. МИР. 1989.
2. Hardy C., Baronet C.N., Tordion G.V. Elastoplastic indentation of a half-space by a rigid sphere.-J.Numerical Methods in Engng., 1971, 3, p.451.
3. Аргатов И.И., Назаров С.А. Метод срашиваемых разложений для задач с малыми зонами контакта // Механика контактных взаимодействия. М. :Физматлит, 2001. С. 73-82
4. Almardonov O.M. "Curved stamp problem" - a collection of scientific and practical theses on the role of talented young people in the development of mathematics, mechanics and computer sciences. Tashkent-2014. Page 6