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Based Solutions Of The Curved Stamp Problem For Elastic Environments

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Abstract: In this article, the issue of the interaction of the stamp with the elastic half-plane based on a certain regularity of the geometric profile is considered, and the case of determining the displacement, deformation components, normal stresses on the border of the stamp, and stresses at the internal points of the half-plane for the case where the boundary conditions are given in displacements is presented.

Keywords: absolute rigid body, displacement, deformation components, boundary conditions, normal stresses.

Introduction

In the study of contact problems, if the size of one of the interacting bodies is sufficiently larger than the other, the second body is considered a half-space, and this formulation of the problem provides an opportunity to find an analytical solution by simplifying the boundary conditions. Plane problems appear as a special case of a deformable half-space. The problem of contact of a semi-elastic plane with a thin sharp plate (concentrated vertical force) was considered by Flaman [1]. The problem is brought to determine the power function satisfying the boundary conditions, and analytical solutions are found. Later, the main results related to the state of stress under vertical and experimental stresses applied to the finite area of the half-plane are given by Timoshenko S.P., D. J. Gooder D. J. Taken by [2]. The main problem here is to find the solution of singular integral equations depending on the type of boundary conditions. It should be noted that the application of integral equations to the theory of elasticity and finding analytical solutions Galin L.A. [3], Muskhilishvili N.I. [4], Nazarov S.A. implemented by [5]. A solution to the problem of a flat stamp with sliding in the contact area was found by L.A. Galin, where the frictional force in sliding in the contact area is considered according to Coulomb's law. Turning to the issues of spatial contact, we can quote Bussinesque [6] and Cherruti [7], the solutions determined by the stress function in determining the stress state of the semi-elastic space under the influence of surface forces. According to Hertz's theory, the stress state of the half-space under distributed stresses at the boundary was studied by Guber [8], who determined the analytical solution of this problem under general polynomial stresses by means of Legendre polynomials. If we take into account the diversity of the boundary conditions of the stamp issue, V.I. Massakovsky and Spence considered a cylindrical stamp considering the frictional force in the contact area and found that the vertical stress in this case is proportional to the displacement in this direction. If we focus on the stress state of the elastic-plastic half-space under the influence of a stamp, the researches carried out in this direction mainly belong to Lockett [9], Richmon [10], K. Johnson [11], Hardy [12]. This was taken into account in scientific research through Tresca's criterion of plasticity, and elastic and plastic areas were determined and analyzed in the semi-elastic space according to the various geometric shapes of the stamp. K. Johnson mainly identified the residual deformations that remain after unloading in areas in the state of plastic deformation. Such a problem has a complex structure in stamps with partial slip, and comes to find approximate solutions, stresses, using special asymptotic methods. Comparing the obtained approximate analytical solutions with the experimental results shows that the analytical solutions are close enough at points closer to the stamp boundary. Since the stamp problem is widely used in manufacturing, such problems have also been seen in linear elastic-viscous media. Based on the generalized viscoelastic model, stamp problems are studied, which mainly come to a system of nonlinear equations. The issue of constructing asymptotic solutions based on the equations obtained in the simple geometric form of the stamp was discussed by V.B. Zelentsov, V.M. It can be seen in the scientific research of Aleksandrov. The constructed asymptotic solutions do not always correspond to the results obtained in practice. The reason for this is the peculiarity of the contact boundary of the stamp with the elastic medium.

Below we will consider the issue of interaction of a stamp with a geometric profile $z = Bx^{n+1}$ and an elastic half-plane, which is common in technical issues.

Accordingly, the displacement and deformation components at the boundary are determined as follows:

$$\overline{u_z} = -Bx^{n+1}, \quad \frac{\overline{\partial u}}{\partial z} = -(n+1)Bx^n, \tag{1}$$

we come to the integral equation.

According to the general theory, there is a relationship between the stress and displacement components in the half-plane $-b \le x \le a$ range as follows, and if the boundary conditions are given in the displacements

$$\begin{aligned} \int_{-b}^{a} \frac{q(s)}{x-s} ds &= \frac{\pi(1-2\nu)}{2(1-\nu)} p(x) - \frac{\pi E}{2(1-\nu^2)} \bar{u}_x'(x), \\ \int_{-b}^{a} \frac{p(s)}{x-s} ds &= \frac{\pi(1-2\nu)}{2(1-\nu)} q(x) - \frac{\pi E}{2(1-\nu^2)} \bar{u}_z'(x). \end{aligned} \tag{2}$$

That is, we will have integral equations.

According to the given displacements at the boundary, these relations form integral equations with respect to the normal, compressive stresses $p(x)$ and $q(x)$. Since *s*=*x* is a special point in the field of integration, these integral equations are called singular integral equations.

If the boundary conditions belong to the 2nd type, then the system of integral equations presented above

$$\int_{-b}^{a} \frac{F(s)}{x-s} ds = g(s),$$

appears. Here, *g(s)* is a function that depends on the displacement at the boundary.

It is an integral equation, the first kind of singular equation, and most contact problems take this form.

General solution of equation (1) in analytical form

$$F(x) = \frac{1}{\pi^2[(x+b)(a-x)]^{1/2}} \int_{-b}^{a} \frac{[(s+b)(a-s)]^{1/2}}{x-s} g(s) ds + \frac{c}{\pi^2[(x+b)(a-x)]^{1/2}},$$

$g(x)$ depending on the function.

If the coordinate origin is placed in the center of the area where the boundary stresses are placed, then the appearance of the function *F(x)* is simplified.

$$F(x) = \frac{1}{\pi^2(a^2-x^2)^{1/2}} \int_{-a}^{a} \frac{(a^2-s^2)^{1/2}}{x-s} g(s) ds + \frac{c}{\pi^2(a^2-x^2)^{1/2}}. \tag{3}$$

In this: $c = \pi \int_{-b}^{a} F(x) dx$.

The above-mentioned integrals consist of characteristic integrals:

$$\int_{-b}^{a}\frac{f(s)ds}{x-s}=\lim_{\varepsilon\to 0}\left[\int_{-b}^{x-\varepsilon}\frac{f(s)ds}{x-s}+\int_{x+\varepsilon}^{a}\frac{f(s)ds}{x-s}\right], \tag{4}$$

is calculated in the form of a limit (Cauchy integral).

This is the solution of the integral equation (if we center the coordinate system)

$$p(x)=\frac{1}{\pi^2(a^2-x^2)^{1/2}}\int_{-a}^{a}\frac{(a^2-s^2)^{1/2}}{x-s}\frac{\pi E\,B\,S^n}{\alpha(1-\nu^2)}ds+\frac{c}{\pi^2(a^2-x^2)^{1/2}},$$

that is, the expression of the normally distributed force on the half-plane surface (in the stamp impression) is derived.

Thus, the distribution of normal stress at the boundary

$$\int_{-1}^{1}\frac{(1-s^2)^{\frac{1}{2}}s^n}{x-s}ds=I_n \tag{5}$$

depends on the integral.

Let's dwell on the value of the given integral.

$$\begin{aligned}
&I_0=\int_{-1}^{1}\frac{(1-s^2)^{1/2}}{x-s}ds=\pi x,\\
&I_1=xI_0-I_0=(x-1)I_0,\\
&\dots\dots\dots\dots\dots\dots\\
&I_n=x^nI_0-x^{n-1}I_0-x^{n-2}I_1-\dots\dots-xI_{n-2}-I_{n-1}.
\end{aligned} \tag{6}$$

Accordingly, the voltage under the stamp looks like this:

$$p(x)=-\frac{E(n+1)\,B\,a^{n+1}}{2(1-\nu^2)\pi}\frac{I_n}{(a^2-x^2)^{1/2}}+\frac{P}{(a^2-x^2)^{1/2}} \tag{7}$$

As a second problem, we consider the case where the contact surface forming the half-plane of the stamp surface is $\overline{u_z}=d\cos x$.

In that case, let's consider $(\overline{u_z})'=-d\sin x$ and the stamp is absolutely smooth ($q(x)=0$)

$$\int_{-a}^{a}\frac{p(s)}{x-s}ds=\frac{-\pi E}{2(1-\nu^2)}d\sin x\ . \tag{8}$$

The solution of this equation is given above

$$p(x)=-\frac{Ed}{2(1-\nu^2)\pi(a^2-x^2)^{1/2}}\int_{-a}^{a}\frac{(a^2-s^2)^{1/2}}{x-s}\sin s ds+\frac{c}{\pi^2(a^2-x^2)^{1/2}} \tag{9}$$

determined by the relationship.

The resulting normal stresses

$$\sigma_x=-\frac{2z}{\pi}\int_{-b}^{a}\frac{p(s)(x-s)^2ds}{[(x-s)^2+z^2]^2},$$

$$\sigma_z = -\frac{2z^3}{\pi}\int_{-b}^{a}\frac{p(s)ds}{[(x-s)^2+z^2]^2},$$

$$\tau_{xz} = -\frac{2z^2}{\pi}\int_{-b}^{a}\frac{p(s)(x-s)ds}{[(x-s)^2+z^2]^2},$$

as we can determine the stresses in the internal points of the half-plane by putting them into relations.

As we know, in the problem of contact of two bodies, one of them is considered as an absolutely rigid body, and when the contact area is small enough, the problem is replaced by the problem of impact with a body with finite dimensions of semi-elastic space. In the problems of interaction of elastic bodies, when the dimensions of the contact surface are much smaller than the dimensions of the bodies, the stresses on the interaction surface do not depend on their configurations at points far from the boundaries of the bodies.

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