

## The method of studying multidimensional random variables related to probability theory and combinatorics

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**Abstract.** In this article discussed that, importance of interdisciplinary integration, some of the requirements and conditions for the introduction of integrated education in the primary grades, and described in detail the general features of the integrated course.

**Key words:** *placements, enterprises, integration, interdisciplinary integration, educational effectiveness, primary education, integration lessons.*

### Introduction

In addition to one-dimensional t.m.s, it is necessary to study quantities whose possible values are determined by 2, 3, ..., n numbers. Such quantities are called two-dimensional, three-dimensional, ..., n-dimensional, respectively.

Let's assume that the  $(\Omega, \mathbf{A}, P)$  defined in the probability space are given.

Suppose that is defined in the probability space  $X_1, X_2, \dots, X_n$  t.m. be given.  $X = (X_1, X_2, \dots, X_n)$  vector is a random vector or n-dimensional etc. is called. An Intuition for each concept is built first before discussing the math. That's why I've created many illustrations to facilitate visually explaining abstract concepts. I've also included links for related topics this post didn't cover. A linkable table of content is provided below, in case you want to jump into specific topics directly. I hope those efforts could offer you a better reading experience.

$F_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n) = P\{X_1 < x_1, X_2 < x_2, \dots, X_n < x_n\}$  n dimensional function is called distribution function of random vector or joint distribution function of etc. For convenience, we write the distribution function in its form, omitting the indices.  $F_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n)$   $X_1, X_2, \dots, X_n$   $F(x_1, x_2, \dots, x_n)$ .

$F(x_1, x_2, \dots, x_n)$  funksiya  $X = (X_1, X_2, \dots, X_n)$  be the distribution function of a random vector. Here are the main properties of the multivariate distribution function:  $F(x_1, x_2, \dots, x_n)$

1.  $\forall x_i : 0 \leq F(x_1, x_2, \dots, x_n) \leq 1$ , that is, the distribution function is bounded.
2.  $F(x_1, x_2, \dots, x_n)$  the function is non-decreasing on any of its arguments and continuous from the left.
3. If any  $x_i \rightarrow +\infty$  if so, then

$$\begin{aligned} \lim_{x_i \rightarrow +\infty} F(x_1, x_2, \dots, x_n) &= F(x_1, \dots, x_{i-1}, \infty, x_{i+1}, \dots, x_n) = \\ &= F_{X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_n}(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) \end{aligned}$$

4. If there is one, then .

The distribution function derived using property 3 (3.1.1)  $X = (X_1, X_2, \dots, X_n)$  is called a marginal (eigen) distribution function. The number of all marginal distribution functions of a random vector is equal to  $\lim_{x_i \rightarrow -\infty} F(x_1, x_2, \dots, x_n) = 0$  bo'lsa, u holda  $k = C_n^1 + C_n^2 + \dots + C_n^{n-1} =$

$$\sum_{n=0}^n C_n^m - C_n^0 - C_n^n = 2^n - 2$$

ga tengdir.

For example, the number of marginal distribution functions of (n=2) two-dimensional random vector is two, which are as follows: .

For simplicity, we will consider the case where n=2, i.e. (X,Y) is a two-dimensional random vector.

$$\begin{aligned} X = (X_1, X_2) & \quad (n=2) & F(x_1, +\infty) = F_1(x_1) = P(X_1 < x_1); \\ F(+\infty, x_2) & = F_2(x_2) = P(X_2 < x_2) \end{aligned}$$

For simplicity, we will consider the case where n=2, i.e. (X,Y) is a two-dimensional random vector.

### 3.2 Two-dimensional discrete random variable and its distribution law

(X,Y) two-dimensional t.m. distribution law

$$p_{ij} = P\{X = x_i, Y = y_j\}; \quad i = \overline{1, n}, \quad j = \overline{1, m}$$

The probability distribution for a discrete random variable X can be represented by a formula, a table, or a graph, which provides  $p(x) = P(X=x)$  for all x. The probability distribution for a discrete random variable assigns nonzero probabilities to only a countable number of distinct x values.

can be given using the formula or in the form of the following table:

Y X	$y_1$	$y_2$	$\dots$	$y_m$
$x_1$	$p_{11}$	$p_{12}$	$\dots$	$p_{1m}$
$x_2$	$p_{21}$	$p_{22}$	$\dots$	$p_{2m}$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$x_n$	$p_{n1}$	$p_{n2}$	$\dots$	$p_{nm}$

$p_{ij}$  the sum of the probabilities is equal to one because  $\{X = x_i, Y = y_j\} \ i = \overline{1, n}, j = \overline{1, m}$  birgalikda bo'lmagan hodisalar to'la

gruppasi tashkil etadi  $\sum_{i=1}^n \sum_{j=1}^m p_{ij} = 1$  the

formula is the distribution law of two-dimensional discrete t.m., and the table is called the joint distribution table.

Given the joint distribution law of two-dimensional discrete t.m. (X,Y), it is possible to find separate (marginal) distribution laws of each component.

Each  $i = \overline{1, n}$  uchun  $\{X = x_i, Y = y_1\}, \{X = x_i, Y = y_2\}, \dots, \{X = x_i, Y = y_m\}$  hodisalar birgalikda bo'lmagani sababli:  $p_{x_i} = P\{X = x_i\} = p_{i1} + p_{i2} + \dots + p_{im}$ . Demak,  $p_{x_i} = P\{X = x_i\} = \sum_{j=1}^m p_{ij}, \ i = \overline{1, n}, \ p_{y_j} = P\{Y = y_j\} = \sum_{i=1}^n p_{ij} \ j = \overline{1, m}$ .

**Example 1.** Two balls are taken at risk from a bowl containing 2 white, 1 black, and 1 blue balls. Among the obtained balls, the number of black balls is X t.m. and the number of blue balls Y t.m. let it be Construct the joint distribution law of (X,Y) two-dimensional t.m. Find the separate distribution laws of X and Y etc.

X t.m. can take values: 0 and 1: values of Y t.m. as well 0 va 1. We calculate the corresponding probabilities:  $p_{11} = P\{X = 0, Y = 0\} = \frac{C_2^2}{C_4^2} = \frac{1}{6}$  (yoki  $\frac{2}{4} \cdot \frac{1}{3} = \frac{1}{6}$ );

$p_{12} = P\{X = 0, Y = 1\} = \frac{C_2^1}{C_4^2} = \frac{2}{6};$   $p_{21} = P\{X = 1, Y = 0\} = \frac{2}{6};$   
 $p_{22} = P\{X = 1, Y = 1\} = \frac{1}{6}.$

(X,Y) The distribution table of the vector looks like this:

Y	0	1
X	0	1

X \ Y	0	1	
0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$
1	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Bu erdan  $P\{X = 0\} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}, \quad P\{X = 1\} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2};$

$P\{Y = 0\} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}, \quad P\{Y = 1\} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$  originates. The separate distribution laws of X and Y etc. will have the following form:

$$\left\{ \begin{array}{l} X : 0, 1 \\ p : \frac{1}{2}, \frac{1}{2} \end{array} \right\}_{va} \left\{ \begin{array}{l} Y : 0, 1 \\ p : \frac{1}{2}, \frac{1}{2} \end{array} \right\}$$

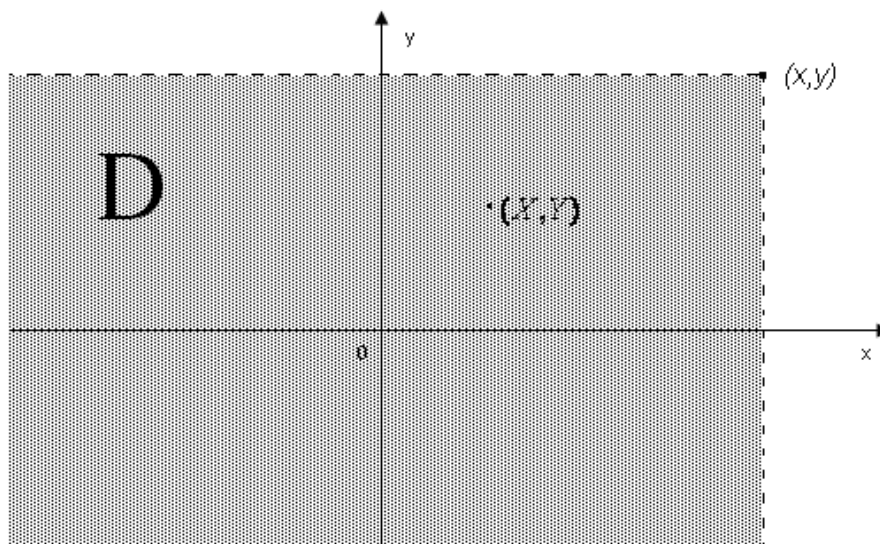
### Distribution function of two-dimensional random variable and its properties

Two-dimensional t.m. we define the distribution function by  $F(x,y)$ .

The distribution function of the two-dimensional  $(X,Y)$  t.m., for each pair of numbers  $x$  and  $y$ , is the function  $F(x,y)$  that determines the joint probability of the events: i.e.

$$F(x, y) = P\{X \leq x, Y \leq y\} = P((X,Y) \in (-\infty, x) \times (-\infty, y) = D)$$

A geometric representation of equality is given in Fig.



$(X,Y)$  two-dimensional discrete t.m. the distribution function is defined by the following sum:

$$F(x, y) = \sum_{x_i < x} \sum_{y_j < y} p_{ij}$$

**Problems for independent solution.**

- Using the given table, find the conditional mean of the sample.

X \ Y	4	4,5	5	5,5		6
8	5	3	-	-		-
10	2	4	5	4		3
13	-	1	1	2		2

- Using the given table, find the conditional mean of the sample.

X \ Y	3	3,5	4	4,5	5
7	5	3	-	-	-
9	2	3	5	3	1
13	-	1	1	2	2

The generalization to Type I standard random variables presented in this section is intended to provide a mathematical support of using such type of standard random variable for representing any arbitrary random variable, independently of its boundaries, as it will be considered in the following sections. In practice, however, it might be more convenient using Type II or Type III standard random variables when they apply, because their boundaries are completely independent of the boundaries. It is also important to mention that the results obtained for continuous variables can be extended to discrete variables by replacing the probability density function with the probability mass function, and replacing integrals with sums.

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