

## **APPROXIMATION OF FUNCTIONS WITH COEFFICIENTS**

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**Annotation:** In mathematics, the approximation of functions with coefficients is a fundamental concept used in various areas such as signal processing, numerical analysis, and machine learning. This article discusses the process of representing a function as a linear combination of basis functions with coefficients and its applications in various fields.

**Keywords:** Approximation, Functions, Coefficients, Basis functions, Signal processing, Numerical analysis, Machine learning.

### **Introduction.**

Approximation of functions with coefficients is a fundamental concept in mathematics and plays a crucial role in various fields such as engineering, physics, and computer science. The idea behind approximation of functions is to find a simpler function that closely represents the behavior of a more complex function. This is particularly useful when dealing with large datasets or when trying to simplify complex mathematical models.

Function is the main object studied in the course of mathematical analysis. In many problems, the complexity of the function related to the calculation of the function (finding its value at a given point) causes great difficulties in such calculations. As a result, the problem of approximating the function with a simpler and easier to calculate function arises.

The expansion of the function in the power series is widely used to approximate it. In this case, replacing the function with the partial sum of the degree series, finding the value of the function at a given point leads to the calculation of the value of the polynomial at this point. The fact that the rank series is simpler in structure, and its partial sum is a simple polynomial, means that the value of the function at a given point can be effectively calculated.

It should also be noted that such a possibility is available only for "good" functions, that is, for functions that have derivatives of any order and satisfy a certain condition. If arbitrary continuous functions are given, the question arises whether it can be approximated using a polynomial. That is, the problem of generalizing the possibility of approximate replacement of a function with a polynomial to the class of continuous functions as analytic functions arises.

The approximation of functions with coefficients involves representing a given function as a linear combination of basis functions with coefficients. This concept is widely used in various fields such as signal processing, numerical analysis, and machine learning. The goal is to find an approximation that closely matches the behavior of the original function while using a simpler representation.

In signal processing, for example, approximating a given signal with coefficients can help reduce its complexity and facilitate analysis. In numerical analysis, approximating mathematical functions with coefficients can help in solving complex equations and performing computations more efficiently. In machine learning, the approximation of functions with coefficients plays a key role in modeling complex data sets and making predictions.

The process of approximating functions with coefficients involves choosing an appropriate set of basis

functions and finding the optimal coefficients that minimize the error between the original function and its approximation. Commonly used basis functions include polynomials, trigonometric functions, and wavelets. Once the basis functions are chosen, techniques such as least squares regression or Fourier series can be used to determine the coefficients that best approximate the original function.

In 1885, the famous German mathematician K. Weierstrass showed that a continuous function can be approximated by polynomials. This fact is expressed by the following theorem.

One common approach to approximating functions is through the use of coefficients. Coefficients are numerical values that are used to represent the magnitude and direction of a particular component in a function. They can be used to approximate various types of functions, including polynomials, trigonometric functions, and exponential functions.

In the context of polynomials, coefficients are used to represent the terms in the polynomial function. For example, consider the polynomial function  $f(x) = ax^2 + bx + c$ . Here, the coefficients  $a$ ,  $b$ , and  $c$  determine the shape and behavior of the parabola represented by the polynomial. By manipulating these coefficients, it is possible to approximate different types of curves and surfaces.

Similarly, in trigonometric functions such as sine and cosine, coefficients are used to represent the amplitudes and frequencies of the waves. By adjusting these coefficients, it is possible to approximate various periodic phenomena such as sound waves or oscillations.

In the case of exponential functions, coefficients are used to represent growth rates and initial values. By adjusting these coefficients, it is possible to approximate exponential growth or decay processes.

One common method for approximating functions with coefficients is through the use of least squares regression. In this approach, a model function with adjustable coefficients is fitted to a set of data points in such a way that it minimizes the sum of squared differences between the model function and the actual data points. This allows for an optimal approximation of the underlying function with a simpler model that can be easily manipulated by adjusting its coefficients.

Another approach for approximating functions with coefficients is through Taylor series expansion. This method involves representing a given function as an infinite sum of terms involving its derivatives evaluated at a specific point. By truncating this series at a certain point and considering only a finite number of terms (which depend on adjustable coefficients), it is possible to obtain an approximation for the original function.

In conclusion, approximation of functions with coefficients is an important tool in mathematics and its applications extend across various disciplines. By manipulating these coefficients, it is possible to closely approximate complex functions with simpler models that can be easily analyzed and manipulated. Whether it's through least squares regression or Taylor series expansion, understanding how to use coefficients for approximation allows for better understanding and manipulation of mathematical models in real-world applications. In mathematics, approximation theory is concerned with how functions can best be approximated with simpler functions, and with quantitatively characterizing the errors introduced thereby. What is meant by best and simpler will depend on the application.

A closely related topic is the approximation of functions by generalized Fourier series, that is, approximations based upon summation of a series of terms based upon orthogonal polynomials.

One problem of particular interest is that of approximating a function in a computer mathematical library, using operations that can be performed on the computer or calculator (e.g. addition and multiplication), such that the result is as close to the actual function as possible. This is typically done with polynomial or rational (ratio of polynomials) approximations.

The objective is to make the approximation as close as possible to the actual function, typically with an accuracy close to that of the underlying computer's floating point arithmetic. This is accomplished by using a polynomial of high degree, and/or narrowing the domain over which the polynomial has to approximate the function. Narrowing the domain can often be done through the use of various addition or scaling formulas for the function being approximated. Modern mathematical libraries often reduce the domain into many tiny segments and use a low-degree polynomial for each segment. Once the domain (typically an interval) and degree of the polynomial are chosen, the polynomial itself is chosen in such a way as to minimize the worst-case error. That is, the goal is to minimize the

maximum value of  $|P(x) - f(x)|$ , where  $P(x)$  is the approximating polynomial,  $f(x)$  is the actual function, and  $x$  varies over the chosen interval. For well-behaved functions, there exists an  $N$ th-degree polynomial that will lead to an error curve that oscillates back and forth between  $+\varepsilon$  and  $-\varepsilon$  a total of  $N+2$  times, giving a worst-case error of  $\varepsilon$ . It is seen that there exists an  $N$ th-degree polynomial that can interpolate  $N+1$  points in a curve. That such a polynomial is always optimal is asserted by the equioscillation theorem. It is possible to make contrived functions  $f(x)$  for which no such polynomial exists, but these occur rarely in practice.

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