AJPDIS GLOBAL RESEARCH NETWORK

**AMERICAN Journal of Public Diplomacy and International Studies**
*Volume 01, Issue 02, 2023 ISSN (E): XXX-XXX*

# The Possibilities of the "Matematica" Package in Solving the Equations of Bernoulli, Clareau and Riccati

**Khonqulov Ulugbek Khursanalievich**
*Associate professor of Fergana State University, (PhD).*

**Madrakhimov Asqarali Ermatovich**
*Associate professor of Fergana State University, (f-m.f.n).*

**Sharapov Bakhrom Abdusalov ugli**
*Master of Fergana State University*

**Abstract:** in the study of differential equations in the article, the disclosure of certain aspects of the "Matematica" package is seen as the main problem. In solving the equations of Bernoulli, Cleero and Riccati, examples are given on the possibilities of the "Matematica" package.

**Keywords:** differential equation, Bernoulli, Cleero, LaGrange and Riccati, computer mathematical systems.

The role of comuter Technologies is felt in the development of students' skills to be able to apply numerical and analytical calculation methods and their orientation to scientific research areas, improving the effectiveness of training. Mathematical Sciences in particular, including the study of differential equations using computer mathematical systems, allow for a conscious break in the subject, in addition to providing practical significance and visualization of the course. Computer programs are an ideal and reliable tool for solving various differential equations, leading to the expansion of mathematical practice. Below we will consider the possibilities of the "Matematica" package in the study of differential equations.

**a)** The following

$$y'+a(x)y=b(x)y^n, \quad n\neq 0,1 \qquad (1)$$

the visual equation is called the Bernoulli equation. Bernoulli's equation is usually defined by $y^{-n}y'+a(x)y^{1-n}=b(x), \ y\neq 0$ by typing in the view, $z=y^{1-n}$ is brought to the linear equation using substitution:

$$\frac{1}{1-n}z'+a(x)z=b(x), \quad z\neq 0,$$

$n>0$ when (1) equation $y=0$ will also have a solution.

**Example 1.** Solve the equation:

a) $4y' - 2y = \dfrac{2x\sin 3x}{y}$ b) $y' + \dfrac{1}{2}y = \dfrac{-x\sin 3x}{2}y^3$

**Solution:** both (a) and (b) equations respectively $n=-1$ and $n=3$ is the Bernoulli equation in which. a) in the equation we introduce the substitution: $w = y^{1-(-1)} = y^2$ as a result $4y' - 2y = \dfrac{2x\sin 3x}{y}$ and $2w' - 2w = 2x\sin(3x)$ a linear equation follows, which we divide by 2 and $w' - w = x\sin(3x)$. We get the equation. We calculate the following integrals:

$$e^{\int (-1)dx} = e^{-x} \text{ va } \int xe^{-x}\sin 3x dx = \frac{(4-5x)\sin 3x - 3(1+5x)\cos 3x}{50e^x}$$

```
In[15]:= stepone = Integrate[x Exp[-x] Sin[3 x], x]
Out[15]= 1/50 e^-x (-3 (1 + 5 x) Cos[3 x] + (4 - 5 x) Sin[3 x])
```

Then $w' - w = x\sin(3x)$ the general solution of the tenlama has the following appearance:

$$w(x) = \frac{(4-5x)\sin 3x - 3(1+5x)\cos 3x}{50e^x} + ce^x$$

```
In[16]:= steptwo = (stepone + c) / Exp[-x]
Out[16]= e^x (c + 1/50 e^-x (-3 (1 + 5 x) Cos[3 x] + (4 - 5 x) Sin[3 x]))
```

If the replacement is taken into account $y(x) = w(x)^{1/2}$. We define it as solutionone:

```
In[17]:= solutionone = steptwo^(1/2) // PowerExpand
Out[17]= e^(x/2) Sqrt[c + 1/50 e^-x (-3 (1 + 5 x) Cos[3 x] + (4 - 5 x) Sin[3 x])]
```

b) for the equation $w = y^{1-3} = y^{-2}$ we enter a replacement. As a result $y' + \dfrac{1}{2}y = \dfrac{-x\sin 3x}{2}y^3$ from the equation $w' - w = x\sin(3x)$ a linear equation follows. The general solution to the equation is as above $w(x) = steptwo$ is found from and $y(x) = w(x)^{-1/2}$ the replacement is taken into account, which *solutiontwo* we define.

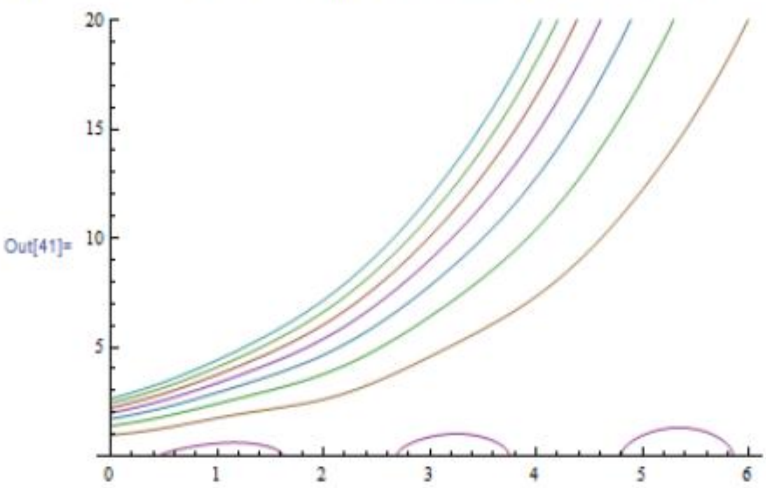
```
In[27]:= solutiontwo = steptwo^(-1/2) // PowerExpand
Out[27]= e^(-x/2) / Sqrt[c + 1/50 e^-x (-3 (1 + 5 x) Cos[3 x] + (4 - 5 x) Sin[3 x])]
```

Solutions *DSolve* it can also be obtained using the function. But we have shown an alternative solution method because *DSolve* does not work properly for all Bernoulli equations. To construct graphs of solutions of equations given for different values of a fixed number in a solution, we construct a list of values of constants and a) and b a table of solutions of equations):

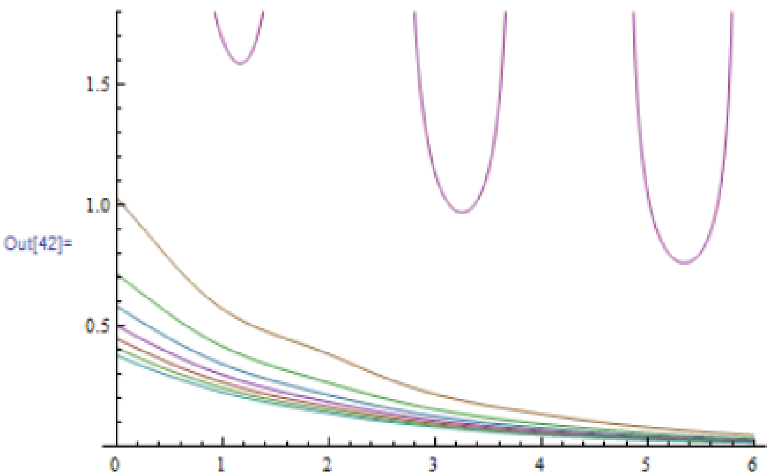
```
In[28]:= cs = Range[-5, 7]
Out[28]= {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7}

In[29]:= tographone = Table[solutionone /. c → cs[[i]],
             {i, 1, 13}];

In[30]:= tographtwo = Table[solutiontwo /. c → cs[[i]],
             {i, 1, 13}];
```

We construct graphs for solutions:

```
In[41]:= Plot[Evaluate[tographone], {x, 0, 6}, PlotRange → {All, {0, 20}}]
Out[41]=
```




```
In[42]:= Plot[Evaluate[tographtwo], {x, 0, 6}, PlotRange → {0, 1.8}]
Out[42]=
```




**b)** This

$$f(xy' - y) = g(y') \quad (2)$$

the visual differential equation is called the Clareau equation. The general solution to such equations is written as: $f(xc - y) = g(c)$, this $c$ an arbitrary constant number.

**Example 1.** Solve the equation: $2(xy'(x)-y(x))^2=(y'(x))^2-y'(x)$.

**Solution:** here for the Clareau equation $f(x)=2x^2+1$ va $g(x)=x^2-x$. $f$ and $g$ we define the functions and derive the following: $f(xy'(x)-y(x))$ and $g(y'(x))$.

```
In[7]:= Clear[f, g, h]
        f[x_] = 2 x^2 + 1;
        g[x_] = x^2 - x;

In[10]:= lhs = f[x y '[x] - y[x]]
Out[10]= 1 + 2 (-y[x] + x y'[x])^2

In[11]:= rhs = g[y '[x]]
Out[11]= -y'[x] + y'[x]^2

In[12]:= lhs == rhs
Out[12]= 1 + 2 (-y[x] + x y'[x])^2 == -y'[x] + y'[x]^2
```

The result obtained confirms that the initial equation is the Clareau equation. In this case, the *Dsolve* function does not work so correctly, that is, it finds the general solution of the equation uncomfortable and cannot find the special solution at all. Therefore, we use the general method of solving *CLAREAU* equations. As mentioned above, the general solution of the unknown form $f(xc-y)=g(c)$ given by the formula, here $c$ is an arbitrary constant number. We find a general solution to the given equation and call it implicit:

```
In[13]:= implicit = f[x c - y] == g[c]
Out[13]= 1 + 2 (c x - y)^2 == -c + c^2
```

*Solve* using the function $y$ we find an exact solution by solving the resulting equation for. We define the result as explicit:

```
In[14]:= explicit = Solve[implicit, y]
Out[14]= {{y → 1/2 (-√2 √(-1 - c + c^2) + 2 c x)},
          {y → 1/2 (√2 √(-1 - c + c^2) + 2 c x)}}
```

$\sqrt{-1-c+c^2}$ root $-1-c+c^2<0$ from unspecified, or approx $c\in(-0,62;1,61)$:

```
In[15]:= Solve[-1 - c + c^2 == 0] // N
Out[15]= {{c → -0.618034}, {c → 1.61803}}
```

Specific solutions *explicit* [[1,1,2] and *explicit*[[2,1,2]] can be obtained with commands:

```
In[16]:= explicit[[1, 1, 2]]
         explicit[[2, 1, 2]]
```

$$\text{Out[16]}= \frac{1}{2}\left(-\sqrt{2}\sqrt{-1-c+c^2}+2\,c\,x\right)$$

$$\text{Out[17]}= \frac{1}{2}\left(\sqrt{2}\sqrt{-1-c+c^2}+2\,c\,x\right)$$

Graphs of the resulting solution $c$ we construct with different values of the constant. For this *Range and Union* using functions -5, -4, -3, -2, -1, 1,2,4,6,8,10,12 and 14 we construct a series of numbers. In this, none of them $(-0,62;1,61)$ does not belong to the interval:

```
In[18]:= cs = Union[Range[-5, -1], Range[2, 14, 2]]
Out[18]= {-5, -4, -3, -2, -1, 2, 4, 6, 8, 10, 12, 14}
```

Then $c$ constantani *cs[i]*, $i$=1,2,…,8 through the value, we construct a table of functions consisting of exact solutions. We open brackets with the *Flatten* command to get a list of functions as a result:

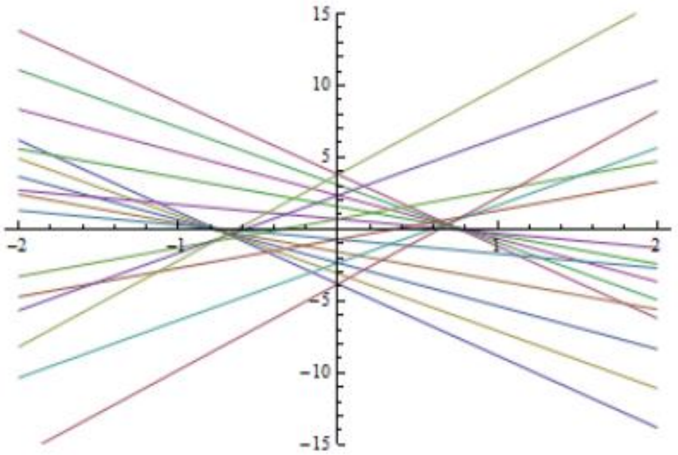
```
In[19]:= tograph = Table[y /. explicit /. c → cs[[i]], {i, 1, 8}] // Flatten
```

$$\text{Out[19]}= \left\{\frac{1}{2}\left(-\sqrt{58}-10x\right),\ \frac{1}{2}\left(\sqrt{58}-10x\right),\ \frac{1}{2}\left(-\sqrt{38}-8x\right),\ \frac{1}{2}\left(\sqrt{38}-8x\right),\ \frac{1}{2}\left(-\sqrt{22}-6x\right),\ \frac{1}{2}\left(\sqrt{22}-6x\right),\ \frac{1}{2}\left(-\sqrt{10}-4x\right),\ \frac{1}{2}\left(\sqrt{10}-4x\right),\ \frac{1}{2}\left(-\sqrt{2}-2x\right),\ \frac{1}{2}\left(\sqrt{2}-2x\right),\ \frac{1}{2}\left(-\sqrt{2}+4x\right),\ \frac{1}{2}\left(\sqrt{2}+4x\right),\ \frac{1}{2}\left(-\sqrt{22}+8x\right),\ \frac{1}{2}\left(\sqrt{22}+8x\right),\ \frac{1}{2}\left(-\sqrt{58}+12x\right),\ \frac{1}{2}\left(\sqrt{58}+12x\right)\right\}$$

Now we draw graphs of these functions in the range [-2,2]. Using *y* the PlotRange command, one can see that the field of detection of is -15 to 15.

```
In[20]:= plotsols = Plot[Evaluate[tograph], {x, -2, 2}, PlotRange → {-15, 15}]
```

Out[20]=




Another solution of a given *CLAREAU* equation that cannot be derived from the general solution is called a special solution. It $x$ with respect to lhs and rhsni obtained by differentiating:

```
In[21]:= dlhs = D[lhs, x]
Out[21]= 4 x (-y[x] + x y'[x]) y''[x]

In[22]:= drhs = D[rhs, x]
Out[22]= -y''[x] + 2 y'[x] y''[x]
```

$4xy''(x)(xy'(x)-y(x)) = y''(x)(2y'(x)-1)$ we get the equation. We will replace it with a form: $y''(x)(1-4xy(x)-2y'(x)+4x^2y'(x))=0$. $y''(x)\neq 0$ since

$1-4xy(x)-2y'(x)+4x^2y'(x)=0$. Left side of the last equation *stepone [[1]]* removed from *stepone* using the command:

```
In[23]:= stepone = Factor[dlhs - drhs]
Out[23]= (1 - 4 x y[x] - 2 y'[x] + 4 x^2 y'[x]) y''[x]

In[24]:= stepone[[1]]
Out[24]= 1 - 4 x y[x] - 2 y'[x] + 4 x^2 y'[x]
```

*DSolve* through the function $1-4xy(x)-2y'(x)+4x^2y'(x)=0$ we solve the equation and solve it *singular* we define.

```
In[25]:= singular = DSolve[stepone[[1]] == 0, y[x], x]
Out[25]= {{y[x] → x/2 + Sqrt[-1 + 2 x^2] C[1]}}
```

*Singular [[1,1,2]]* by order $\frac{x}{2}+\sqrt{-1+2x^2C[1]}$ we get the answer and mark it as Sol.

```
In[26]:= sol = singular[[1, 1, 2]]
Out[26]= x/2 + Sqrt[-1 + 2 x^2] C[1]
```

$2((xy'(x)-y(x))^2=(y'(x))^2-y'(x)$ under the condition of satisfying the equation *C[1]* we find the constant. *Sols* $x$ we calculate the first and second derivatives with respect to and calculate the results respectively *dsol* and *d2sol* we define:

```
In[27]:= dsol = D[singular[[1, 1, 2]], x]
```

$$\text{Out[27]=}\ \frac{1}{2}+\frac{2\,x\,C[1]}{\sqrt{-1+2\,x^2}}$$

```
In[28]:= d2sol = D[singular[[1, 1, 2]], {x, 2}]
```

$$\text{Out[28]=}\ \left(-\frac{4\,x^2}{\left(-1+2\,x^2\right)^{3/2}}+\frac{2}{\sqrt{-1+2\,x^2}}\right)C[1]$$

To the given equation and respectively *sol*, *dsol*, *d2sol* expressions $y[x]$, $y'[x]$ and $y''[x]$ we will replace. The resulting equation is *C[1]* solving with respect to the constant, one obtains its solutions as roots:

```
In[29]:= tosolve = lhs == rhs /. {y[x] → sol, y'[x] → dsol, y''[x] → d2sol}
```

$$\text{Out[29]=}\ 1+2\left(-\frac{x}{2}-\sqrt{-1+2\,x^2}\,C[1]+x\left(\frac{1}{2}+\frac{2\,x\,C[1]}{\sqrt{-1+2\,x^2}}\right)\right)^2==$$
$$-\frac{1}{2}-\frac{2\,x\,C[1]}{\sqrt{-1+2\,x^2}}+\left(\frac{1}{2}+\frac{2\,x\,C[1]}{\sqrt{-1+2\,x^2}}\right)^2$$

```
In[30]:= roots = Simplify[Solve[tosolve, C[1]]]
```

$$\text{Out[30]=}\ \left\{\left\{C[1]\to-\frac{\sqrt{\frac{5}{2}}}{2}\right\},\left\{C[1]\to\frac{\sqrt{\frac{5}{2}}}{2}\right\}\right\}$$

A special solution to a given equation is $y(x)=\frac{x}{2}\pm\sqrt{\frac{5}{8}}\sqrt{2x^2-1}.$

```
In[36]:= singgraphs = singular /. roots // Flatten
```

$$\text{Out[36]=}\ \left\{y[x]\to\frac{x}{2}-\frac{1}{2}\sqrt{\frac{5}{2}}\sqrt{-1+2\,x^2},\ y[x]\to\frac{x}{2}+\frac{1}{2}\sqrt{\frac{5}{2}}\sqrt{-1+2\,x^2}\right\}$$

Singgraphs of the resulting solutions *[[1,2]]* and Singgraphs *[[2,2]]* we can distinguish from:

```
In[37]:= singgraphs[[1, 2]]
```

$$\text{Out[37]=}\ \frac{x}{2}-\frac{1}{2}\sqrt{\frac{5}{2}}\sqrt{-1+2\,x^2}$$

```
In[38]:= singgraphs[[2, 2]]
```

$$\text{Out[38]=}\ \frac{x}{2}+\frac{1}{2}\sqrt{\frac{5}{2}}\sqrt{-1+2\,x^2}$$

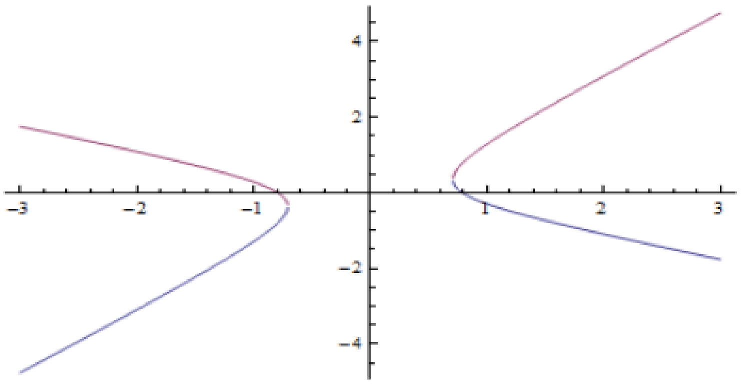
Special solutions $2x^2-1<0$ from unspecified, or (-0,708; 0,708).

```
In[39]:= Solve[-1 + 2 x^2 == 0] // N
Out[39]= {{x → -0.707107}, {x → 0.707107}}
```

(-3; -0,708) and (0,708; 3) we draw graphs of special solutions at intervals. We use the *PlotRange* option to show the entire graph and the AxesOrigin option to cut coordinate axes at (0,0):

```
In[43]:= pone = Plot[Evaluate[{singgraphs[[1, 2]], singgraphs[[2, 2]]}],
            {x, -3, -0.708}];
         ptwo = Plot[Evaluate[{singgraphs[[1, 2]], singgraphs[[2, 2]]}],
            {x, 0.708, 3}];

In[48]:= Show[pone, ptwo, PlotRange → All, AxesOrigin → {0, 0}]
```

Out[48]=




**c)** This

$$y'+a(x)y+b(x)y^2=c(x)$$

the visual equation is called the *Riccati* equation. If one renders the *Riccati* equation $y=y_1(x)$ if the private solution is known, then it can be $y=y_1+z$ the substitution can be brought to *Bernoulli's* equation. The private solution to *Riccati's* equation is in some cases $c(x)$ - using the appearance of the Free term $y=ax+b,\quad y=\frac{a}{x}$ can be found by selection in the view.

**Example 1.** Riccati equation $y(0)=0$ solve in the initial condition and [-0,5;1] draw a graph of solutions in the interval:

$$y'+(x^4+x^2+1)y^2-\frac{2(1-x+x^2-2x^3+x^4)}{1+x^2+x^4}y+\frac{1}{x^4+x^2+1}=0$$

**Solution:** In The Equation *a(x)*, *b(x)* va *c(x)* we find the expressions:

```
In[12]:= a[x_] = x^4 + x^2 + 1;
         b[x_] = -2 (1 - x + x^2 - 2 x^3 + x^4) / (1 + x^2 + x^4);
         c[x_] = 1 / (x^4 + x^2 + 1);
```

$y(x)=\frac{w'(x)}{w(x)}\ \frac{1}{a(x)}$ by substituting, we obtain the 2-order equation:

```
In[15]:=
eqone = y'[x] + a[x] y[x]^2 + b[x] y[x] + c[x] == 0 /.
  {y[x] -> w'[x]/(w[x] a[x]), y'[x] -> D[w'[x]/(w[x] a[x]), x]}

Out[15]= 1/(1 + x^2 + x^4) - ((2 x + 4 x^3) w'[x])/((1 + x^2 + x^4)^2 w[x]) -
  (2 (1 - x + x^2 - 2 x^3 + x^4) w'[x])/((1 + x^2 + x^4)^2 w[x]) + w''[x]/((1 + x^2 + x^4) w[x]) == 0
```

*w(x)a(x)* we simplify the expression:

```
In[16]:= eqtwo = Simplify[eqone[[1]] w[x] a[x]] == 0
Out[16]= w[x] - 2 w'[x] + w''[x] == 0
```

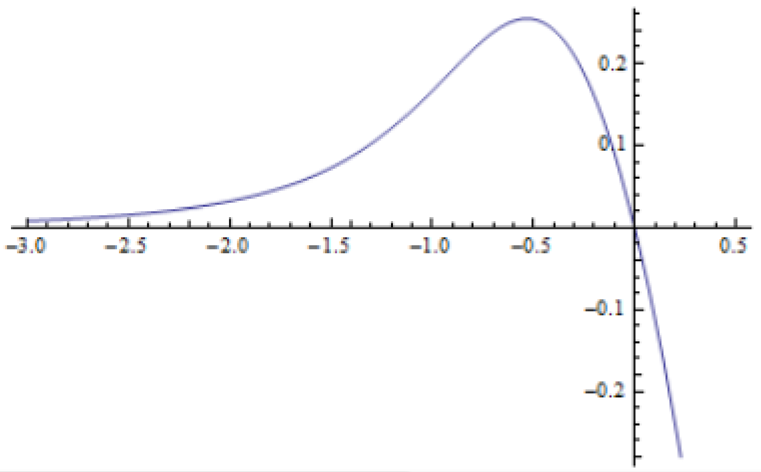
As a result, *Riccati* becomes a 2-order equation: $w''(x) - 2w'(x) + w(x) = 0$ from *w(x)* we find. *NDSolve* using the function, [-0,5; 1] in the range, the initial condition is y(0)=0 we find an approximate solution to this *Riccati* equation, which is:

```
In[31]:= approx = NDSolve[{y'[x] + a[x] y[x]^2 + b[x] y[x] + c[x] == 0, y[0] == 0},
   y[x], {x, -3, .5}]
Out[31]= {{y[x] → InterpolatingFunction[{{-3., 0.5}}, <>][x]}}
```

We draw a graph for the resulting solution:

```
In[32]:= Plot[y[x] /. approx, {x, -3, .5}]
```

Out[32]=




**Literature used:**


1. Khankulov, U. K. (2017). Description of Methodical System of Teaching Elements of Stochastics Line Mathematics Using Computer Technologies. *Eastern European Scientific Journal*, (6).
2. Хонкулов, У. Х. (2018). ON THE SELECTION PRINCIPLES OF EDUCATIONAL CONTENT OF STOCHASTIC ELEMENTS OF MATHEMATICS. *Наука и мир*, 2(4), 56-57.

3. Хонкулов, У. Х. (2015). О применениях компьютерных технологий в обучении элементов стохастики. *Региональные проблемы преобразования экономики*, (8 (58)), 76-82.

4. Zharov, V. K. (2012). Formanov Sh. K., Honkulov Yu. Kh. On the statistical method in a pedagogical experiment in the context of a modern educational process. *Bulletin of the Moscow State Regional University. Series Pedagogy*, (3), 104-110.