

## Non-Standard Methods for Solving Trigonometric Inequalities

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**Abstract:** This article describes non-standard ususes for solving trigonometric equations. It presents the method of sectors and the method of concentric circles from non-standard methods of solving trigonometric inequalities based on the principle of the genetic approach.

**Keywords:** trigonometric issue, method of concentric circles, method of sectors, genetic approach.

**Introduction:** The purpose of teaching trigonometry in specialized schools (educational, educational, practical) is determined by the implementation of trigonometry in solving practical issues related to various fields and disciplines, as well as the application of trigonometric functions. Trigonometry is distinguished by great importance in theory and practice, in particular, by the fact that it is an excellent computational apparatus in solving various problems of geometry, by the means of trigonometric functions it is convenient to express mathematical concepts such as periodicity, monotony, pointing accuracy, delimitation, pairwise rigidity, bias and convex, continuity in a clear, visual and understandable way [2,3,4]. In particular, the use of non-standard methods in solving inequalities of the trigonometric equation extends the field of application of trigonometry. Below we give several non-standard methods for solving trigonometric inequalities.

### Research Methodology:

a) *Sectors method in solving trigonometric inequalities.*

*Let's consider the method of sectors when solving trigonometric inequalities.*

$\frac{P(x)}{Q(x)} > 0, (< 0, \geq 0, \leq 0)$  in the form (where  $P(x)$  and  $Q(x)$  rationally introduced

trigonometric functions) the solution of rational trigonometric inequalities is almost identical to the solution of rational inequalities. It is convenient to solve rational inequalities by the method of intervals on the number axis. As its analogue  $\sin x$  and  $\cos x$  ( $T = 2\pi$ ) for the trigonometric circle,  $\operatorname{tg} x$  and  $\operatorname{ctg} x$  ( $T = \pi$ ) for the solution of the rational trigonometric inequality in the semicircle, the method of sectors is considered [5,6,7].

1.  $\frac{P(\sin x, \cos x)}{Q(\sin x, \cos x)} > 0$  ( $< 0, \geq 0, \leq 0$ ) solving form inequalities.

The numerator and denominator of an expression given in the interval method are  $(x - x_0)$  whose factors correspond to it  $x_0$  when passing through the point, the exchange of signals is determined, and the corresponding intervals are obtained depending on the signal of the inequality. In the sectoral method, however, each factor of the numerator and denominator of a given expression is  $(f(x) - a)$  in form. Here  $f(x) - \sin x$  or  $\cos x$ ,  $-1 < a < 1$  is one of the functions, in the trigonometric circle  $x_1$  and  $x_2$  divides two sectors corresponding to the corners. These corners are  $(f(x_1) = f(x_2) = a)$  is equivalent to  $x_1$  from  $x_2$  in the transition to  $(f(x) - a)$  the signal of the expression is determined.

The following should be borne in mind:

a)  $(\sin x - a)$  and  $(\cos x - a)$  form factors  $|a| > 1$  while  $x$  does not change the signal at all values of. Therefore, such multipliers are discarded. If  $a > 1$  if, by changing the inequality signal to the opposite, the factors are discarded.

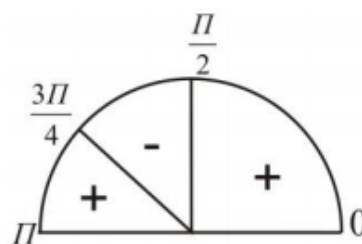
b)  $(\sin x \pm 1)$  and  $(\cos x \pm 1)$  form factors are also discarded. Moreover, if these are multiples of the denominator, then  $\sin x \neq \pm 1$  and  $\cos x \neq \pm 1$  will be thrown under the condition.  $(\sin x - 1)$  or  $(\cos x - 1)$  when form factors are discarded, the inequality signal changes to the opposite.

2.  $\frac{P(\text{tg}x, \text{ctg}x)}{Q(\text{tg}x, \text{ctg}x)} > 0$  ( $< 0, \geq 0, \leq 0$ ) solving form inequalities.

$\text{tg}x$  or  $\text{ctg}x$  was one of the functions  $f(x)$  of the function  $(f(x) - a)$  each factor of the form is a trigonometric semicircle  $-\frac{\pi}{2} < x < \frac{\pi}{2}$  (or  $0 < x < \pi$ ) at one  $x_0$  fits the angle, or  $f(x_0) = a$ . This  $x_0$  when passing a point  $(f(x_0) - a)$  the signal of the multiplier alternates. In addition,  $\text{tg}x$  function  $x = \pm \frac{\pi}{2}$  is not defined at, and will have a different signal on the left and right of these points. Similar  $\text{ctg}x$  function  $x = 0$  va  $x = \pi$  at undefined and to the right of these points there are different symbols.

1- **example.** Solve the inequality:  $\text{ctg}^2 x + \text{ctg}x > 0$ .

**Solution:**  $\text{ctg}x(\text{ctg}x + 1) > 0$ . Trigonometric half  $0 < x < \pi$  in the circle  $\text{ctg}x = 0$  the equation is given by  $x_1 = \frac{\pi}{2}$  angle,  $\text{ctg}x = -1$  the equation is given by  $x_1 = \frac{3\pi}{4}$  the angle is suitable. They divide the semicircle into three sectors, and their in each  $y = \text{ctg}x(\text{ctg}x + 1)$  the function saves the signal and exchanges hints in each sector (1-picture).



1-picture

$0 < x < \frac{\pi}{2}$  in the sector  $\text{ctgx}(\text{ctgx}+1) > 0$ . And in the rest, the hint alternates.

$y = \text{ctgx}(\text{ctgx}+1)$  period of function  $T = \pi$ .  $\text{ctg}^2x + \text{ctgx} > 0$  since it is positive-signal sectors we choose.

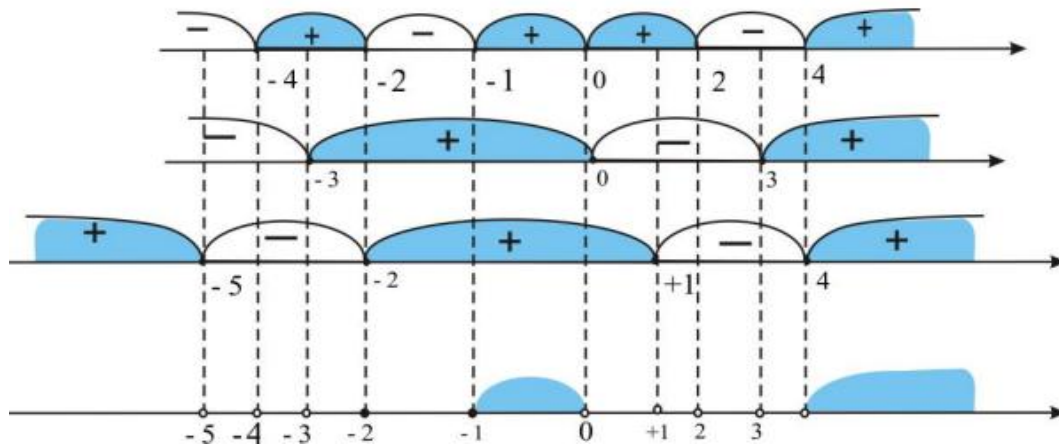
**Answer:** 
$$\begin{cases} \pi n < x < \frac{\pi}{2} + \pi n \\ \frac{3}{4}\pi + \pi n < x < \pi + \pi n \end{cases}, \quad n \in \mathbb{Z}.$$

b) method of concentric circles solving a system of *trigonometric* inequalities.

This method is compared to the parallel number axis method when solving a system of rational inequalities. Let's see the following example.

$$\begin{cases} \frac{(x+2)^3(x+1)(x-4)}{(x+4)x^2(x-2)} \geq 0, \\ \frac{x}{(x+3)(x-3)} \geq 0, \\ \frac{(x-1)(x+2)}{(x+5)(x-4)} \geq 0. \end{cases}$$

If the system of inequalities is solved on a single number of axes, the image presents difficulties in isolating the solution, it is much easier to isolate the solution if solved on the parallel number axis (2-picture).



**2-picture. Solving inequalities on the parallel number axis.**

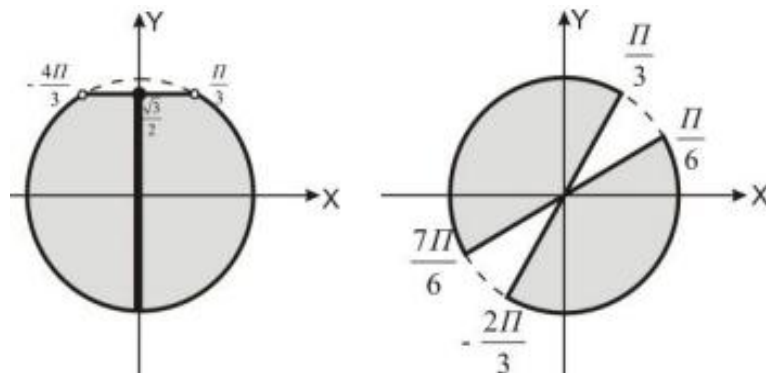
**Answer:**  $\{-2\} \cup [-1; 0) \cup (4; +\infty)$ .

**1- example.** Solve the system of inequalities:

$$\begin{cases} \cos x < -\frac{1}{2}, \\ \sin 2x < \frac{\sqrt{3}}{2}, \\ \operatorname{tg} x \geq -1 \end{cases}$$

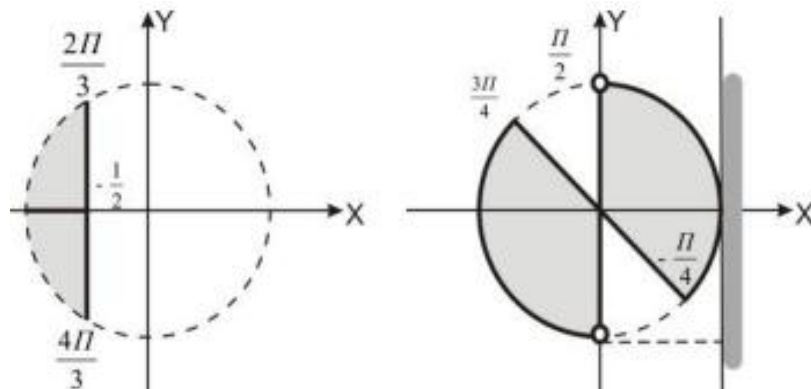
**Solution.** We solve each inequality separately in the unit circle.

1) Argument  $2x$  is  $\sin 2x < \frac{\sqrt{3}}{2}$  solving the inequality (3-picture).



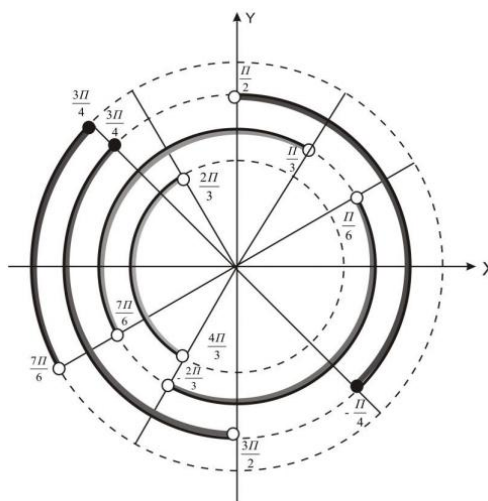
**3-picture.**

2)  $\cos x < -\frac{1}{2}$  and  $\operatorname{tg} x \geq -1$  we solve for inequalities (4-picture).



**4-picture.**

Now  $x$  for arguments, we draw concentric circles. We draw a circle corresponding to the solution of the first inequality and we are the shitrch, then we draw a circle with a larger radius and we are the shitrch according to the solution of the second, then we draw a circle and a base circle for the third inequality. From the center of the circle, we pass rays through the ends of the arcs so that they cross all circles. The result is a solution in the base circle (5-picture).



5-picture.

Answer:  $\left[ \frac{3\pi}{4} + 2\pi k; \frac{7\pi}{6} + 2\pi k \right], k \in \mathbb{Z}.$

**Example 2.** Solve the inequality:  $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x > 0.$

**Solution.**  $1 - \cos 2\alpha = 2\sin^2 \alpha$  using the formula, we substitute the form:

$$1 - \cos\left(2x + \frac{\pi}{2}\right) + \sqrt{3}\cos 2x > 0, \quad -\cos\left(2x + \frac{\pi}{2}\right) + \sqrt{3}\cos 2x > -1,$$

$$\sin 2x + \sqrt{3}\cos 2x > -1, \quad \frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x > -\frac{1}{2},$$

$$\sin \frac{\pi}{6}\sin 2x + \cos \frac{\pi}{6}\cos 2x > -\frac{1}{2}, \quad \cos\left(2x - \frac{\pi}{6}\right) > -\frac{1}{2}.$$

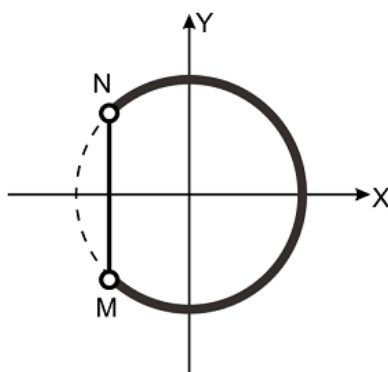
We solve for the last inequality.  $t = 2x - \frac{\pi}{6}$  let's enter a mark, as a result:  $\cos t > -\frac{1}{2}$ . We

describe the solution in the unit circle (6-picture):

$$-\frac{2\pi}{3} + 2\pi k < t < \frac{2\pi}{3} + 2\pi k.$$

$x$  let's go back to the variable:

$$-\frac{2\pi}{3} + 2\pi k < 2x - \frac{\pi}{6} < \frac{2\pi}{3} + 2\pi k \Rightarrow -\frac{\pi}{4} + \pi k < x < \frac{5\pi}{12} + \pi k, \quad k \in \mathbb{Z}.$$



6-picture.

**Conclusion/Recommendations:** In conclusion, high requirements are imposed on the knowledge, skills and qualifications of students of a specialized school, of which, first of all, it is required to have a high level of admission indicators to higher educational institutions, while maintaining a deep and fluent mastery of educational material in order to achieve positive results in Olympiads conducted in different disciplines. In this case, the application of non-standard methods opens the way for the development of methodological issues of teaching mathematics and the solution of problems in its teaching.

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