AMERICAN Journal of Engineering, Mechanics and Architecture
Volume 2, Issue 3, 2024 ISSN (E): 2993-2637

The Flat Problem of the Theory of Elasticity and its Foundations

Almardonov Oybek Makhmatkulovich
Karshi Engineering Economic Institute, 180100, Karshi, Uzbekistan

A body is said to be in a state of plane deformation, if the displacements of the points of the body are parallel to one plane and do not depend on the coordinate in the direction perpendicular to the plane (of the points). If the 0xy plane is taken as the deformation plane, then

$$u = u(x, y), \quad \varepsilon_z = \frac{\partial w}{\partial z} = 0,$$

$$v = v(x, y), \quad \gamma_{xz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad (1.10)$$

$$w = 0, \quad \gamma_{xz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0.$$

In turn $\tau_{xz} = \tau_{yz} = 0$ γz=yz=0 and σ_z z while $\varepsilon_z = \frac{1}{E}(\sigma_z - \mu(\sigma_x + \sigma_y))$, z=1E(z-μ(x+y)) is determined from the relation: $\sigma_z = \mu(\sigma_x + \sigma_y)$.

Thus, in the case of plane deformation, the problem of the theory of elasticity becomes much simpler, and the spatial problem comes to a two-dimensional problem.

Indeed,

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0 \text{ ва } z = 0 \text{ z=xz=yz=0 ва Z=0}$$

In this case, two of Nave's equations remain, and three $\sigma_x, \sigma_y, \tau_{xy}$ x , y , xy the equations remain with respect to the stress components.

Accordingly, Nave's equations take the following form:

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0, \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial y} + Y = 0. \end{cases} \quad (1.11)$$

And for boundary conditions:

$$\sigma_x l + \tau_{xy} m - \overline{X} = 0,$$

$$\tau_{yx} m + \sigma_x l - \overline{Y} = 0 \quad (1.12)$$

relationships must be fulfilled.

In turn, the Cauchy relation and the Saint-Venant equation for the planar problem

$$\varepsilon_x = \frac{\partial u}{\partial x},\ \varepsilon_x = \frac{\partial v}{\partial y},\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \quad (1.13)$$

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}, \quad (1.14)$$

appears. The conditions for sharing the remaining 5 deformations are self-fulfilled. For the connection between strain and stress components (Hooke's law) $\varepsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y)$,

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x), \quad (1.15)$$

$$\gamma_{xy} = \frac{2(1+\mu)}{E}\tau_{xy},$$

relationships are appropriate.

To write the system of equations in stresses, we use Hooke's law to condition the deformations together.

After normal operations

$$\frac{\partial^2 \sigma_y}{\partial x^2} + \frac{\partial^2 \sigma_x}{\partial y^2} - \mu\frac{\partial^2 \sigma_x}{\partial x^2} - \mu\frac{\partial^2 \sigma_y}{\partial y^2} + 2(1+\mu)\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = 0,$$

and using the equilibrium equations:

$$\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = -\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = -\frac{\partial^2 \sigma_y}{\partial y^2},$$

$$\frac{\partial^2 \sigma_y}{\partial x^2} + \frac{\partial^2 \sigma_x}{\partial y^2} - \mu\frac{\partial^2 \sigma_x}{\partial x^2} - \mu\frac{\partial^2 \sigma_y}{\partial y^2} + (1+\mu)(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}) = 0,$$

we form the equation with respect to the stresses and after simplification

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})(\sigma_x + \sigma_y) = 0,$$

or

$$\nabla^2\ (\sigma_x + \sigma_y) = 0,\ \nabla^2 = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}). \quad (1.16)$$

we come to the harmonic equation.

In this case, the planar problem satisfies equilibrium equations, boundary conditions, and harmonic equations $\sigma_x, \sigma_y, \tau_{xy}$ x , y , xy comes to find the voltage components. Such an issue φ is rendered convenient to solve by introducing a stress function.

$$\sigma_x = \frac{\partial^2 \varphi}{\partial y^2}, \quad \sigma_x = \frac{\partial^2 \varphi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}. \quad (1.17)$$

In this case, the equations of equilibrium become real and from the condition of sharing the deformations together

$$\nabla^2 \nabla^2 \varphi = 0, \quad (1.18)$$

a biharmonic equation is derived. In this

$$\nabla^2 \nabla^2 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}.$$

If volume forces U has potential, then for volume forces

X=-∂U∂x , Y=-∂U∂y $X = -\frac{\partial U}{\partial x}, \quad Y = -\frac{\partial U}{\partial y}$

and the voltage function

$$\sigma_x - U = \frac{\partial^2 \varphi}{\partial y^2},$$

$$\sigma_y - U = \frac{\partial^2 \varphi}{\partial x^2},$$

$$\tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y},$$

if we enter through the relations, from the condition that the deformations are shared:

$$\nabla^2 \nabla^2 \varphi + (1-\mu)\nabla^2 U = 0, \quad (1.19)$$

we get the equation

Thus, the plane problem of the theory of elasticity is reduced to the determination of the stress function satisfying the equation and the boundary conditions.

Below we consider the flat problem of the theory of elasticity in the polar coordinate system. Because the next "contact" issues are studied mainly in polar coordinates.

Between Cartesian coordinates and polar coordinates

$$r^2 = x^2 + y^2, \quad \theta = arctg\frac{y}{x},$$

according to the connection and between the private derivations

$$2r = \frac{\partial r}{\partial x} = 2x, \; 2r = \frac{\partial r}{\partial y} = 2y,$$

based on relationships:

$$\frac{\partial r}{\partial x} = \frac{x}{r} = \cos\theta, \quad \frac{\partial r}{\partial y} = \frac{y}{r} = \sin\theta, \quad \frac{\partial \theta}{\partial x} = \frac{\frac{\partial}{\partial x}(\frac{y}{2})}{1+\frac{y^2}{x^2}} = -\frac{y}{r^2} = -\frac{\sin\theta}{r} \quad \frac{\partial \theta}{\partial r} = \frac{\cos\theta}{r}.$$

In this case, Nave's equations look like this.

$$\begin{cases} \frac{\partial \theta_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta}\frac{1}{r} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \\ \frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial \sigma_\theta}{\partial \theta}\frac{1}{r} + 2\frac{\tau_{r\theta}}{r} = 0, \end{cases} \qquad (1.20)$$

Deformations corresponding to linear and angular changes are:

$$\begin{cases} \varepsilon_r = \frac{\partial u}{\partial r}, \qquad \varepsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\mathrm{u}}{\mathrm{r}}, \\ \gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial \mathrm{v}}{\partial \mathrm{r}} - \frac{\mathrm{v}}{\mathrm{r}}, \end{cases}$$

For Hooke's law in turn

$$\sigma_r = \frac{E}{1-\mu^2}(\varepsilon_r + \mu\varepsilon_\theta), \quad \sigma_\theta = \frac{E}{1-\mu^2}(\varepsilon_\theta + \mu\varepsilon_r),$$

$$\tau_{r\theta} = \frac{E}{2(1+\mu)}\gamma_{r\theta},$$

relationships are appropriate. Now let's consider the biharmonic equation in the polar coordinate system.

The voltage function in the polar coordinate system is as follows

$$\sigma_r = \frac{1}{r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial r^2}, \quad \sigma_\theta = \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \varphi}{\partial r^2}, \quad \tau_{r\theta} = \frac{1}{r^2}\frac{\partial \varphi}{\partial \theta} - \frac{1}{r}\frac{\partial^2 \varphi}{\partial r^2},$$

is entered in the form, then the equilibrium equations become concrete. Now let's dwell on the conditions for coexistence of deformation in polar coordinates.

$\varphi = \varphi(r, \theta)$ considered as a function of polar coordinates

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial \varphi}{\partial \theta}\frac{\partial \theta}{\partial x} = \frac{\partial \varphi}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial \varphi}{\partial \theta}\sin\theta,$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{d}{dx}\left(\frac{\partial \varphi}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial \varphi}{\partial \theta}\sin\theta\right)\frac{\partial r}{\partial x} + \frac{d}{d\theta}\left(\frac{\partial \varphi}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial \varphi}{\partial \theta}\sin\theta\right)\frac{\partial \theta}{\partial x} =$$

$$\frac{\partial^2 \varphi}{\partial r^2}\cos^2\theta - \frac{2}{r}\frac{\partial^2 \varphi}{\partial \theta \partial r}\sin\theta\cos\theta + \frac{\partial \varphi}{\partial r}\frac{\sin^2\theta}{r} + \frac{2}{r^2}\frac{\partial \varphi}{\partial r}\sin\theta\cos\theta + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}\sin^2\theta,$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial^2 \varphi}{\partial r^2}\sin^2\theta + \frac{2}{r}\frac{\partial^2 \varphi}{\partial \theta \partial r}\sin\theta\cos\theta + \frac{\partial \varphi}{\partial r}\frac{\cos^2\theta}{r} - \frac{2}{r^2}\frac{\partial \varphi}{\partial r}\sin\theta\cos\theta + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}\cos^2\theta,$$

and accordingly, for the Laplace operator

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2},$$

while the proper and biharmonic equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2}\right) = 0 \qquad (1.21)$$

Remains to be seen.

Thus, the flat problem of the theory of elasticity comes to determine the biharmonic equation and the stress function satisfying the boundary conditions in the polar coordinate system.

Thus, the main equations of the theory of elasticity consist of the Nave equations related to the internal points of the body, Hooke's law of the connection between the strain tensor and the stress tensor, the condition for the coexistence of deformation and a set of boundary conditions. In this case, it is required to find the solution of the system of equations with respect to stress and strain tensor components and displacements. In particular, i.e., smooth problems come to find the solution of the biharmonic equation satisfying the boundary conditions.

References:

1. Djonson K. “Mexanika kontaktnogo vzaimodeystviya”. Moskva. MIR. 1989.
2. Hardy C., Baronet C.N., Tordion G.V. Elastoplastic identation of a half-space by a rigid sphere.-J.Numerical Methods in Engng., 1971, 3, p.451.
3. Argatov I.I., Nazarov S.A. Metod sraщivayeмых razlojeniy dlya zadach s malыmi zonami kontakta // Mexanika kontaktnых vzaimodeystviya. M. :Fizmatlit, 2001. S. 73-82
4. Makhmatkulovich, Almardonov Oybek. "Basic Methods and Tools for Solving Contact Issues." *American Journal of Engineering, Mechanics and Architecture (2993-2637)* 1.10 (2023): 292-294.
5. Makhmatkulovich A. O. Fundamentals of Determining the Stressed State of Semi-Elastic Space under the Action of a Force Applied to One Point When Solving Contact Problems //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2023. – Т. 1. – №. 10. – С. 269-272.
6. Almardonov O. M. Based Solutions Of The Curved Stamp Problem For Elastic Environments //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2023. – Т. 1. – №. 9. – С. 67-70.
7. Suvonovich, Khalilov Mukhtor, Yusupov Rustam Eshpulatovich, and Almardonov Oybek Makhmatkulovich. "UNIVERSAL MOUNTED SPRAYER FOR PEST AND DISEASE CONTROL IN ORCHARDS AND VINEYARDS." *Galaxy International Interdisciplinary Research Journal* 9.05 (2021): 349-352.
8. Irgashev D. B. et al. High softening performance indicators of plug-softener //IOP Conference Series: Earth and Environmental Science. – IOP Publishing, 2023. – Т. 1284. – №. 1. – С. 012032.
9. Irgashev D. B. Analysis of Machines Providing Liquid Fertilizer to the Root System of Orchard and Vine Seedlings //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2023. – Т. 1. – №. 10. – С. 356-362.
10. Irgashev D. B., Buriyev M. Analysis of the Coils Used in Soil Drilling //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2023. – Т. 1. – №. 10. – С. 302-308.
11. Irgashev D. B. Basing the Constructional Parameters of the Plug-Softener that Works between the Garden Rows //Journal of Research in Innovative Teaching and Inclusive Learning. – 2023. – Т. 1. – №. 3. – С. 14-19.
12. Irgashev D. B. CALCULATION OF THE STRENGTH OF WINGED JOINTS //JOURNAL OF THEORY, MATHEMATICS AND PHYSICS. – 2023. – Т. 2. – №. 7. – С. 1-6.
13. Irgashev D. B. COUPLINGS USED IN MECHANICAL ENGINEERING AND THEIR IMPORTANCE //JOURNAL OF ENGINEERING, MECHANICS AND MODERN ARCHITECTURE. – 2023. – Т. 2. – №. 7. – С. 1-10.

14. Mamatov F. M. et al. BOG ‘QATOR ORALARINI ISHLOV BERADIGAN QIYA USTUNLI ISHCHI ORGANLARNI PARAMETRLARNI NAZARIY ASOSLASH //JOURNAL OF ENGINEERING, MECHANICS AND MODERN ARCHITECTURE. – 2023. – T. 2. – №. 5. – C. 42-45.
15. Mamatov F. et al. Justification of the bottom softening parameters of working organ with a sloping column //E3S Web of Conferences. – EDP Sciences, 2023. – T. 434. – C. 03010.
16. Irgashev D. THEORETICAL JUSTIFICATION OF THE LONGITUDINAL DISTANCE OF A PLUG-SOFTENER THAT WORKS WITHOUT TURNING THE SOIL BETWEEN GARDEN ROWS //Science and innovation. – 2023. – T. 2. – №. D11. – C. 482-487.
17. Begmurodvich, Irgashev Dilmurod. "Development and Problems of Vineyard Network in Uzbekistan." *Web of Synergy: International Interdisciplinary Research Journal* 2, no. 1 (2023): 441-448.
18. Irgashev D. B., Buriyev M. Analysis of the Coils Used in Soil Drilling //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2023. – T. 1. – №. 10. – C. 302-308.
19. Bekmurodovich I. D. TECHNICAL CLASSIFICATION OF MACHINES THAT TILL THE SOIL BETWEEN ROWS OF VINEYARDS //Uzbek Scholar Journal. – 2022. – T. 10. – C. 369-379.
20. Irgashev D. БОҒ ҚАТОР ОРАЛАРИГА ИШЛОВ БЕРИШДА ТАКОМИЛЛАШГАН ПЛУГ-ЮМШАТКИЧНИНГ ТЕХНИК ТАХЛИЛИ //Science and innovation. – 2022. – T. 1. – №. D7. – C. 330-336
21. Bekmurodovich I. D. TECHNICAL CLASSIFICATION OF MACHINES THAT TILL THE SOIL BETWEEN ROWS OF VINEYARDS //Uzbek Scholar Journal. – 2022. – T. 10. – C. 369-379.
22. Irgashev D. B., Buriyev M. Theoretical Justification of the Forces Generated on the Cylinder Surface of a Double Row Cat //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2024. – T. 2. – №. 2. – C. 157-162.
23. Irgashev D. B. AGROTECHNICAL REQUIREMENTS FOR DEEP TILLAGE WITHOUT TURNING THE SOIL //НАУЧНОЕ ОБЕСПЕЧЕНИЕ УСТОЙЧИВОГО РАЗВИТИЯ АГРОПРОМЫШЛЕННОГО КОМПЛЕКСА. – 2021. – С. 591-594.
24. Иргашев Д. Б. и др. БОҒ ҚАТОР ОРАЛАРИНИ ТЕКИС АҒДАРМАСДАН ИШЛОВ БЕРАДИГАН ҚИЯ УСТУНЛИ ЮМШАТКИЧНИ РАМА КОНСТУРУКЦИЯСИДА ЖОЙЛАШИШ АСОСЛАШ //BARQARORLIK VA YETAKCHI TADQIQOTLAR ONLAYN ILMIY JURNALI. – 2022. – T. 2. – №. 11. – C. 138-146.
25. Irgashev D. B., Ganiev B. G. Based On Longitudinal Distance Of Sloped Column Working Bodies Working Between Garden Rows //American Journal of Engineering, Mechanics and Architecture (2993-2637). – 2023. – T. 1. – №. 9. – C. 48-51.
26. Irgashev D. B., Safarov A. A. TUPROQQA AGDARGICHSIZ ISHLOV BERADIGAN ISHCHI ORGAN KONSTRUKSIYLARI VA ULARGA QO’YILAGAN TALABLLAR //Analysis of International Sciences. – 2023. – T. 1. – №. 3. – C. 6-12.