AMERICAN Journal of Engineering, Mechanics and Architecture
Volume 2, Issue 2, 2024 ISSN (E): 2993-2637

Methods for Calculating Torsional Vibrations of Shafts

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Abstract: The article considers the processes of free and forced torsional vibrations of a shaft with a single mass, and provides basic definitions. The differential equations of free and forced torsional vibrations of a shaft with one mass are derived and their solution is given.

Keywords: shaft, torsional oscillations, free oscillations, forced oscillations, differential equation, stresses, twists, critical velocities, resonance, amplitudes, torque, harmonics, damping forces.

Introduction

Modern methods of calculating the torsional vibrations of the shafts of djinn machines make it possible to determine, with a greater or lesser degree of accuracy, those additional twisting stresses that will occur in the shaft when djinn machines are operating at any of its critical speeds, under conditions of precise torsional resonance. The criterion for the presence of this resonance is considered to be the coincidence of the angular η_h frequency of the resonating torque harmonics on the shaft of the washing machines with one of the angular frequencies ω_0 of the natural torsional vibrations of the shaft of the investigated installation.

Relationship:

$$\gamma = \frac{\eta_h}{\omega_0},$$

Hereinafter referred to as the tuning coefficient, it receives a value of $\gamma = 1$ under resonance conditions.

An external sign of the resonant nature of the oscillatory process is usually an intensive increase in the amplitudes of forced oscillations. Such an increase, or development of amplitudes, continues until the energy input in the oscillating system due to the excitation work performed by resonating harmonic torques is equal to its consumption due to the presence of damping forces in the installation. The maximum resonant amplitude of the oscillation is determined from the equality of these works for each oscillation cycle. It is this limiting amplitude that is given by the currently known methods of calculating the resonant vibrations of the shafts of washing machines [3.15, 17, 18].

But such "favorable" (for the development of vibrations) ideal conditions for a sufficiently long operation of the installation at exactly the pm value of the tuning coefficient $\gamma = 1$ are essentially never implemented in practice, since in actual power plants from washing machines both η_h and ω_0 are always variable.

The angular frequency η_h of the harmonic torque $M_h \sin \eta_h t$ is related to the angular velocity of rotation of the shaft η and the order h of this moment by the ratio:

$$\eta_h = h\eta$$

Thus, η_h is constant in time only under the condition $\eta = const.$

In general, in the dynamic study of installations from washing machines, it is necessary to distinguish:

a) the average (imaginary or conditional) angular velocity η_0 of the rotation of the shaft of the jinny machines;

b) the instantaneous angular velocity η_s of the rotation of the genie shaft, when this shaft, together with all the shafts of the installation, is considered as an absolutely rigid (twisting) body;

c) the true instantaneous speed η_i of rotation of any particular section of the shaft.

The average angular velocity η_0 is determined by the formula:

$$h_0 = \frac{\pi n}{30}, sek^{-1}$$

After a minute, the number of n washing machines, measured, total counters. The speed of η_0 is usually constant with a constant load on the washing machines and its constant adjustment.

The instantaneous velocity η_i is continuously changing due to the inequality between the instantaneous value of the variable torque on the shaft of the washing machines and the moment of the external load.A specific value of η_i for each moment of time can be found, for example, using the well-known Wittenbauer method. Fluctuations in this speed depend on the nature of the total diagram of tangential forces on the shaft of the washing machines and the law of change of the moment of the external load along the angle of rotation of the shaft.

Obviously, η_i can also be called the instantaneous rotation speed of the shaft of the jinn machines, i.e., all its sections and all the masses rotating with it.

The true instantaneous angular velocity η_i due to torsional vibrations of the shaft line is not only variable in time, but also varies along the length of the installation shaft. The velocities η_i are analytically determined only by a detailed calculation of the torsional vibrations of the shaft line and the construction of a torsion diagram for each section of the shaft we are interested in separately.

The relative magnitude of the velocity fluctuations ns and η_i during one cycle of the shaft operation is characterized, respectively, by the expressions:

$$\delta_s = \frac{\eta_{smax} - \eta_{smin}}{\eta_0},$$

$$\delta_i = \frac{\eta_{imax} - \eta_{imin}}{\eta_0}$$

The value δ_s is the "rigid" degree of unevenness, or, as it is often called, the stem of the unevenness of the flywheel, is calculated using a well–known method, assuming the absolute rigidity of the shaft of the twisting installation. Naturally, δ_s is the same for all shaft sections.

The value δ_i, to which we will assign the name of the "elastic" (or true) degree of unevenness, is no longer the same for different shaft sections. Experimentally, δ_i can be found using torsion mapping of the section (or, more precisely, the section) of the shaft of interest to us.

The "hard" degree of unevenness δ_s corresponding to the so-called nodeless vibrations of the shaft enters δ_i through vibrations caused by the main orders of torque harmonics, far from resonance, if by this δ_i we mean the full degree of unevenness due to the effect on the shaft of the entire harmonic spectrum, the torque of the washing machines and the moment of external load.

All generally accepted methods for calculating shaft vibrations, especially forced vibrations, involve replacing all masses variably involved in the oscillatory process with rotating masses with a constant moment of inertia. In fact, as is known, the moment of inertia of the diamond saws reduced to the radius is α periodic function of the angle of rotation of the shaft. Accordingly, the frequency of natural oscillations of the shaft line changes periodically in the range from some from ω_{max} to ω_{min}. Here it is advisable to introduce the concept of an "internal" degree of unevenness:

$$\delta_0 = \frac{\omega_{max} - \omega_{min}}{\omega_0}$$

and characterizing the range of oscillations (pulsations) of the natural oscillation frequency relative to its average value ω_0. The frequency of these natural frequency pulsations depends both on the rotation speed of the genie shaft and on the number of genie saws inserted on it and their relative position.

In contrast to the "internal" degree of unevenness δ_0, which is entirely conditioned by the basic properties of the oscillating system itself, it is possible to assign the values δ_s and δ_i the name of "external" degrees of unevenness (respectively "rigid" and "elastic"), since the latter are essentially determined by external forces acting on the system.

In any installation of the shafts of washing machines, continuous vibrations of η and ω_0 (characterized by degrees of inequality δ_s, δ_0 and δ_i) all the time deviate the system from that state of a stationary oscillatory process, which is only considered by the usual theory of harmonic vibrations. A similar phenomenon occurs when the installation is running at any of its critical speeds.

It seems quite obvious that such a permanent violation of the resonant oscillatory process in the shaft due to external and internal unevenness (δ_s, δ_0 and δ_i) and excludes the possibility of developing those resonant amplitudes and stresses that are calculated by the usual method assuming $\gamma = const = 1$.

With an increase in δ, additional, so-called apparent or dynamic damping increases in the oscillating system. At the same time, unlike actual damping (for example, internal friction on the shaft, friction in bearings, etc.), dynamic damping is not associated, at least mainly, with energy dissipation.

However, the issue of dynamic damping of torsional vibrations has hardly been developed. Analytical accounting of the degree of unevenness (external and internal) by the magnitude of the resonant amplitudes by the magnitude of the resonant vibration amplitudes of the shafts can currently be performed only using the well-known Mansi formula [14].

This formula, attractive for its simplicity and clarity, is quoted in many works on the theory of vibrations by the outfit of the authors [1, 4, 10, 11, 12] and is recommended for use in computational practice. But there is no consensus on the interpretation of the Mansi formula and in assessing the degree of its accuracy. Its fundamental grounds cannot be considered convincing. Moreover, a critical analysis of the Mansi formula and some related views on the effect of dynamic damping force us to recognize it as clearly unsatisfactory [5].

Its application can lead to serious errors not only in the quantitative, but also in the qualitative assessment of those dynamic processes that play out in the shaft line of the power plant of washing machines.

The purpose of this work is to carry out a detailed analysis of the influence of the uneven rotation of the shaft of jinny machines on the nature and development of resonant and nonresonant vibrations in it, as well as to establish the true effect of dynamic damping caused by this unevenness.

When studying shaft vibration, it is necessary to keep in mind the fundamental difference between the "external" (δ_s,δ_i) and "internal" (δ_0) degrees of unevenness in the installations of washing machines.

The "internal" unevenness ($\delta_0 > 0$) caused by periodic changes in the moments of inertia of the reduced gin masses is entirely related to the main characteristics of the installation. Here, the reduction system, at least at not too small δ_0, loses the properties of a harmonic resonator, and its vibrations acquire a quasi-harmonic character.

The "external" illegality (δ_s,δ_i), that is, the uneven rotation of the shaft, does not make such significant changes to the oscillating system, which, as in the ideal case (δ_s= 0), under some of the restrictions listed below, can usually be described by linear differential equations with constant coefficients. Thus, when studying vibrations of installations with constant reduced masses ($\delta_0 = 0$) having $\delta_0 = 0$ (and, to a certain extent, $\delta_i > 0$), we still have the opportunity to use the entire basic apparatus of the theory of harmonic oscillations with its spectral approach to analyzing the effect of complex periodic excitation.

In real oscillatory systems, such as the installations of laser machines, there are usually simultaneously $\delta_0 > 0$ and $\delta_s > 0$ or, more precisely, $\delta_0 > 0$ and $\delta_i > 0$. However, the study of vibrations in these conditions presents exceptional difficulties.

We will limit ourselves here only to analyzing the case $\delta_0 = 0$, in order to identify in its pure form, the effect of the presence of $\delta_s > 0$ (or $\delta_i > 0$) in a linear system, and thereby establish the specific value of the uneven rotation of the shaft as a factor of dynamic damping of torsional vibrations in the installation of washing machines.

According to the location, each saw blade creates a variable torque on the shaft M_{tor}, the value of which, at any angle of rotation of the shaft α, can be determined by conventional methods, according to the indicator diagram. According to the data obtained in this way, the M_{tor} curve is built in the function α, which has, depending on the position of the shaft, a period of 2π or 4 π. According to the data obtained in this way, the M_{tor} curve is built in the function α, which has, depending on the position of the shaft, a period of 2π or 4 π.

The harmonic analysis of this curve, which, as a rule, always satisfies Dirichlet conditions, allows us to represent the torque of one saw blade as a trigonometric series:

$$M_{kr} = M_0 + \sum_{h=1}^{\infty} A_h sinh\,\alpha + \sum_{h=1}^{\infty} B_h cosh\,\alpha = M_0 + \sum_{h=1}^{\infty} M_h sin(h\alpha + \varepsilon_h), \quad (1)$$

(h =1, 2, 3, …)

M_0 – the constant component of the torque M_{tor},

M_h – the amplitude of the harmonic moment (or, in short, the amplitude of the harmonic) *h* – the order determined by the formula:

$$M_h = \sqrt{A_h^2 + B_h^2}$$

ε_h - the phase angle of this harmonic, calculated by the formula:

$$\varepsilon_h = arctg\, {}^{R_h}/_{A_h}$$

and characterizing the position of the vector M_h relative to the position of the shaft at the moment corresponding to α = 0.

In the future, for the sake of simplicity of all calculations, we will consider only the position of the shaft.

The angle of rotation of the shaft α is related to time t by an obvious ratio:

$$\alpha = \int_0^t \eta dt, \qquad (2)$$

where η is the instantaneous angular velocity of the shaft rotation. Now from (1):

$$M_{kr} = M_0 + \sum_{h=1}^{\infty} A_h sin\left(h\int_0^t \eta dt\right) + \sum_{h=1}^{\infty} B_h cos\left(h\int_0^t \eta dt\right) =$$

$$M_0 + \sum_{h=1}^{\infty} M_h sin\left(\left(h\int_0^t \eta dt\right) + \varepsilon_h\right), \qquad (3)$$

In the generally accepted methods for calculating shaft vibrations, a direct proportionality between α and t is assumed:

$$\alpha = \eta_0 t, \qquad (4)$$

where η_0 obviously represents some average angular velocity of rotation of the shaft over the entire considered time interval, that is, at least for the duration of one cycle of the shaft's working process.

The stand (4) in (1) gives the usual spectral representation of the torque M_{kr} in the form of a sum of elementary moments (harmonics), each of which is considered as a simple harmonic function of time

$$M_{kr} = M_0 + \sum_{h=1}^{\infty} A_h sinh\,\eta_0\mathrm{t} + \sum_{h=1}^{\infty} B_h cosh\eta_0\mathrm{t} = M_0 + \sum_{h=1}^{\infty} M_h sin(h\eta_0\mathrm{t} + \varepsilon_h), \qquad (5)$$

Here, all M_h, A_h, B_h, ε_h εh are considered constant, independent of time. But this assumption is the less true, the less stable the mode of operation of the washing machines and, as will be shown below, the greater the unevenness of the shaft rotation.

At constant load on the shaft of the washing machines, the accuracy of the series (5), as a characteristic of the true (i.e. harmonic time) torque spectrum M_{kr}, is entirely determined by the errors of the ratio (4), on the basis of which, as a result of a purely geometric (i.e. without taking into account the time factor) harmonic analysis of the curve $M_{kr} = f_0(\alpha)$ time is entered and a number (5) is obtained.

It is obvious that in all real practice tasks:

$$\eta \neq const$$

А, следовательно, ряд (1) переходит в (3), где каждый слагаемое вида:

$$(M_{kr}) = M_h(h\int \eta dt + \varepsilon_h)\ , \qquad (6)$$

It turns out to be often a very complex function of time *t*.

Thus, the "geometric" harmonic analysis of the curve $M_{kr} = f_0(\alpha)$, the results of which are recorded in the form of an ordinary series (1), still does not answer the question of the specific characteristics of the harmonic torque spectrum M_{kr}, the components of which, under the conditions of the study of forced vibrations of linear systems with constant parameters, must be pure there are many harmonic functions of time. To directly obtain these multi-harmonic torque components, it would be necessary to conduct a harmonic analysis of the M_{kr} curve, constructed as a function of time *t*.

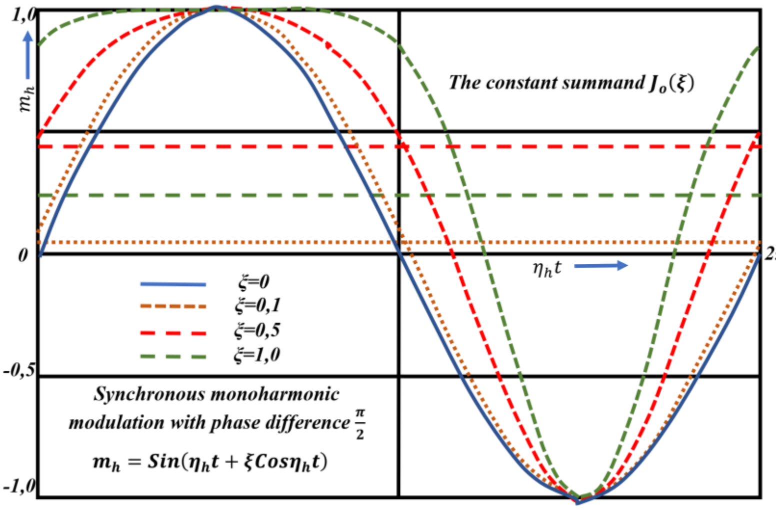

Fig.1. Synchronous modulation with phase $\alpha = \pi/_2$ the amplitude of the moment M_h value increases with increasing modulation amplitudes

However, the construction of such a curve, at least at the beginning of calculations of shaft vibrations, presents insurmountable difficulties, since the dependence of η_0 on *t*. is not sufficiently known in advance.

In addition, the curve $M_{kr} = f_0(\alpha)$ has a well-defined physical meaning, since the main thermal processes occurring in the working process of the collision of the saw blade with the raw material are not so much an explicit function of time, that is, the angle of rotation of the shaft. Finally, it simply does not make sense to refuse to use the results of large computational work on the harmonic analysis of typical torque diagrams conducted by a number of researchers [3,10, 15] and presented either in the form of graphs or tables, giving specific values of the harmonic coefficients $\frac{M_{kr}}{F_r}$ (where F is the area of the diamond saw). The use of such graphs facilitates the work of the designer, since it frees him from the need to perform painstaking calculations on the harmonic analysis of the M_{kr} curve every time.

Thus, it is still advisable to start our study with a series (1), each component of which:

$$(M_{kr})=M_h sin(h\alpha + \varepsilon_h)=M_h sin\left(h \int_0^t \eta dt + \varepsilon_h\right), \qquad (7)$$

It represents a simple harmonic function α and, at the same time, a complex modeled (as usual, under modulation, the introduction of any special, special periodic deviations into a purely sinusoidal oscillatory process) function of time *t*.

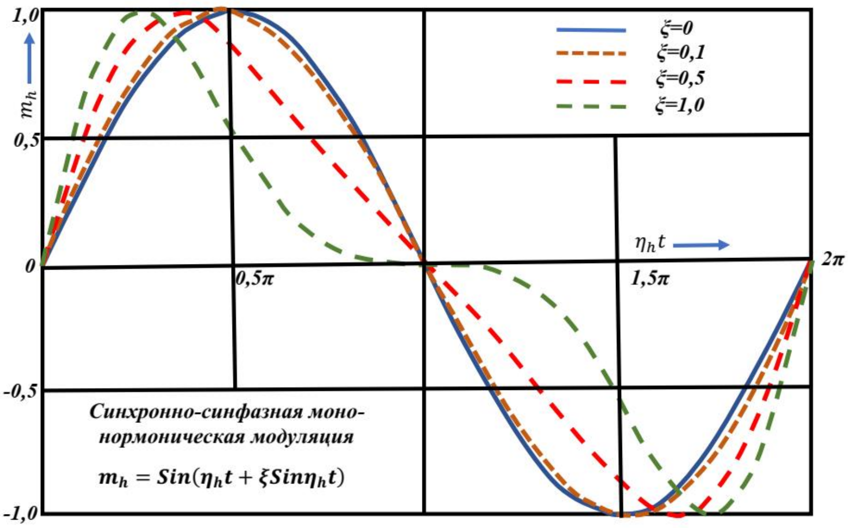

Fig.2. Synchronous-synphase modulation, when the modulation frequency is equal to the carrier frequency ε_h

In accordance with the accepted definitions and terminology (6), we will distinguish the following main types of modulation. Here:

$\eta = \eta_0 = const,$

$$\varepsilon_h = const$$

That is

$$\alpha = h\eta_0 t$$

and

$(M_h) ==M_h sin(h\eta_0 t + \varepsilon_h)$, (8)

where the amplitude of the moment M_h is variable in time. The degree of deviation (M_h) from a purely sinusoidal function t is determined by the nature of the changes in t. amplitude M_h.

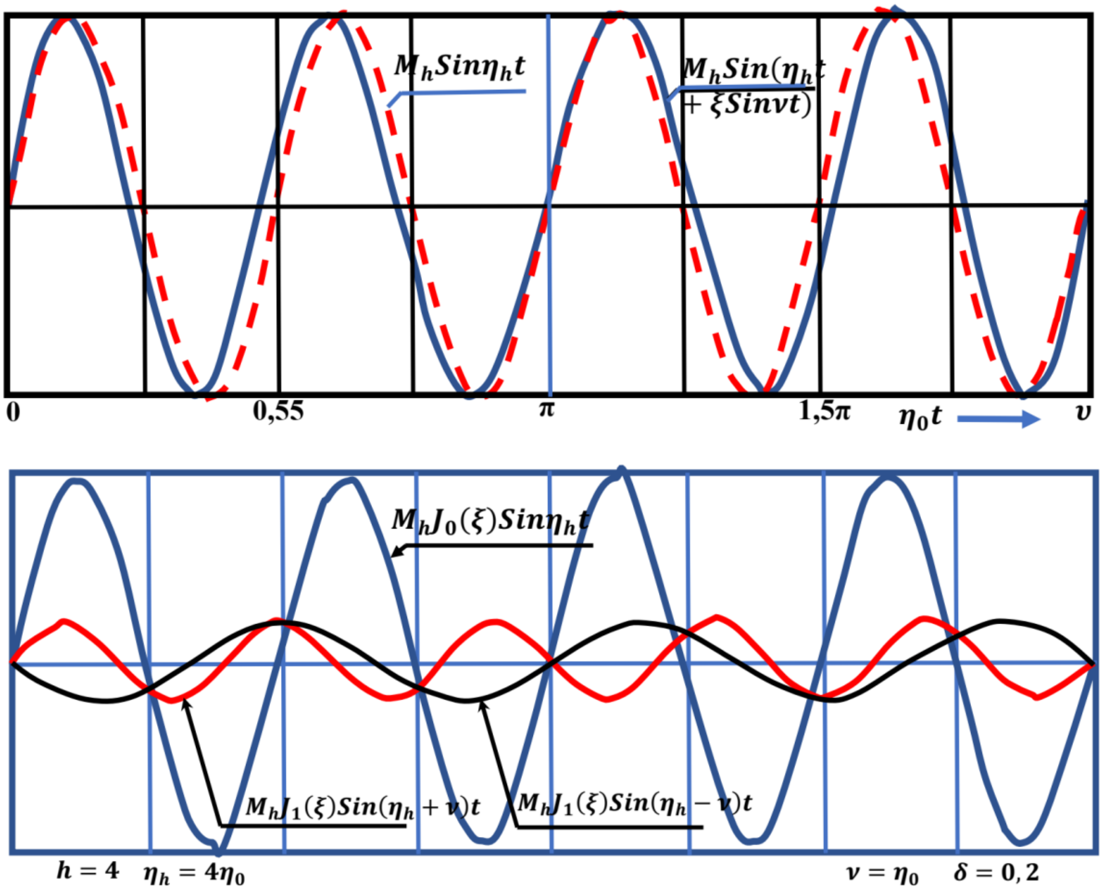

Fig.3. Of the sinusoidal function, the amplitude of the moment M_h by time changes *t*

Phase modulation, or phase modulation, characterized by conditions:

$\eta = \eta_0 = const,$

$M_h = const,$

and the phase angle ε_h - varies depending on time. It is easy to show that phase modulation is similar to frequency modulation. Frequency modulation through the variable rotation speed of the vector M_h, obviously, inevitably leads to changes in ε_h and therefore is simultaneously phase modulation. In view of this, it is not particularly necessary to consider phase modulation.

Mixed modulation, - when M_h and η are time variables. Such differentiation of the M_{kr} components is advisable already with the usual analysis of shaft vibrations of installations operating in a wide range of revolutions. It is especially necessary in our study, since cyclic fluctuations in the angular velocity of the shaft rotation change the magnitude of the inertia forces accordingly.

At $\eta = const$, the inertial harmonics are easily calculated analytically, according to the well-known formulas (5). This allows us to use a torque curve based on an indicator diagram without taking into account inertia forces for conducting harmonic analysis.But, of course, in the final results of this analysis, that is, in all M_h and ε_h included in series (1), in the future it is necessary to take into account the amplitudes and phases of inertial harmonics accordingly.

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