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Fundamentals of Determining the Stressed State of Semi-Elastic Space under the Action of a Force Applied to One Point When Solving Contact Problems

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Abstract: This article presents the definition of the stressed state of a semi-elastic space under the action of a force applied to one point, and also presents methods for determining the components of the stress tensor and the components of the strain tensor.

Keywords: polar coordinate system, deformation, stress, displacement components, boundary conditions, yield strength.

Determining the stressed state of a semi-elastic space under the action of a force applied to one point is an integral part of contact problems. Many scientists have conducted research on these issues. For example, in this problem Flamand expressed the stress function in the form of the $\varphi(r,\theta) = Ar\theta\sin\theta$ polar coordinate system and determined the components of the stress tensor and the components of the strain tensor.

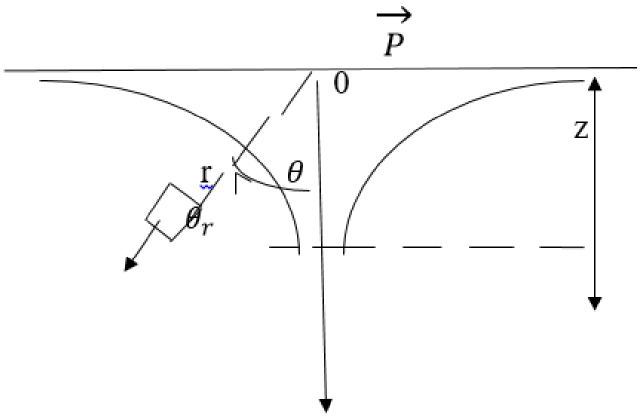

Fig.-1

In this case, A - is constant and the voltages are determined as follows (Fig-1).

$$\sigma_r = \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 2A\cos\theta/r,$$

$$\sigma_\theta = \frac{1}{r^2}\frac{\partial^2\varphi}{\partial r^2} = 0, \qquad (1)$$

$$\sigma_{r\theta} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varphi}{\partial\theta^2}\right) = 0.$$

To determine the constant A, we use the sum of the projections of radial stresses acting on a circle of arbitrary radius r onto the vertical axis 0z:

$$P = -\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \sigma_r \cos\theta d\theta r = -2A\int_0^{2\pi} \cos^2\theta d\theta = -A\pi \,. \, (2)$$

From this

$$\sigma_r = -\frac{2P}{\pi}\frac{\cos\theta}{r} \,. \, (3)$$

According to the result obtained, the radial stress s_r along a circle with diameter d passing through the coordinate system point 0 remains unchanged (Fig. 1). For stress tensor components in the Cartesian coordinate system

$$\sigma_x = \sigma_r \sin^2\theta = -\frac{2P}{\pi}\frac{x^2 z}{(x^2+z^2)^2},$$

$$\sigma_z = \sigma_r \cos^2\theta = -\frac{2P}{\pi}\frac{z^3}{(x^2+z^2)^2}, \qquad (4)$$

$$\tau_{zx} = \sigma_r \sin\theta\cos\theta = -\frac{2P}{\pi}\frac{xz^2}{(x^2+z^2)^2},$$

we will have a relationship.

Now let's calculate the components of the deformation tensor.

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) = \frac{1-\nu^2}{E}\frac{2P}{\pi}\frac{\cos\theta}{r},$$

$$\varepsilon_\theta = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} = \frac{\nu(1+\nu)}{E}\frac{2P}{\pi}\frac{\cos\theta}{r}, \qquad (5)$$

$$\gamma_{r\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}.$$

Let's integrate the resulting relations

$$u_r = \frac{1-\nu^2}{\pi E}2P\cos\theta\ln r - \frac{(1-2\nu)(1+\nu)}{\pi E}P\theta\sin\theta + c_1\sin\theta + c_2\cos\theta,$$

$$u_\theta = \frac{1-\nu^2}{\pi E}2P\sin\theta\ln r + \frac{\nu(1+\nu)}{\pi E}2P\theta\sin\theta - \frac{(1-2\nu)(1+\nu)}{\pi E}2P\theta\cos\theta + \quad (6)$$

$$+\frac{(1-2\nu)(1+\nu)}{\pi E}P\sin\theta + c_1\cos\theta - c_2\sin\theta + c_3 r.$$

Oz since points on the minor axis $U_\theta(0) = 0$ move only along this axis, it follows that $c_1 = c_3 = 0$. $\theta = \pm\frac{\pi}{2}$ in for movements corresponding to angles

$$\bar{u}_r(\frac{\pi}{2}) = \bar{u}_r(-\frac{\pi}{2}) = -\frac{(1-2\nu)(1+\nu)}{\pi E}P$$

$$\bar{u}_\theta(\frac{\pi}{2}) = \bar{\bar{u}}_\theta(-\frac{\pi}{2}) = -\frac{1-\nu^2}{E}2P\ln r + c \qquad (7)$$

the result is reasonable. Results on the state of deformation of a semi-elastic medium under the action of an experimental force can be obtained as a result of rotating the medium through an angle 90^0 relative to the normal force, i.e. calculation of the pole angle from the vertical force.

$$\sigma_r = -\frac{2Q}{\pi}\frac{\cos\theta}{r}, \quad \sigma_\theta = \tau_{r\theta} = 0. \quad (8)$$

And the coordinate is in the *Oxz* plane

$$\sigma_x = -\frac{2Q}{\pi}\frac{x^3}{(x^2+z^2)^2},$$
$$\sigma_z = -\frac{2Q}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \quad (9)$$
$$\tau_{xz} = -\frac{2Q}{\pi}\frac{x^2 z}{(x^2+z^2)^2}.$$

Stressed state of a half-plane under the action of uniformly distributed vertical forces.

Below we will consider the stressed state of a medium consisting of a half-plane under the action of uniformly distributed vertical forces.

Respectively:

$$p(x) = p; \quad q(x) = 0, \quad -a < x < a \quad (10)$$

For the stress tensor components, substituting the constant p into the above formulas

$$\sigma_x = -\frac{P}{2\pi}\left[2(\theta_1-\theta_2)+(\sin 2\theta_1 - \sin 2\theta_2)\right],$$
$$\sigma_z = -\frac{P}{2\pi}\left[2(\theta_1-\theta_2)-(\sin 2\theta_1 - \sin 2\theta_2)\right], \quad (11)$$
$$\tau_{xz} = \frac{P}{2\pi}(\cos 2\theta_1 - \cos 2\theta_2).$$

$tg\theta_{1,2} = \frac{z}{x \pm a}$. If you enter the designation $\alpha = \theta_1 - \theta_2$, then the main voltages are determined as follows:

$$\sigma_{1,2} = (p/\pi)(\alpha \pm \sin\alpha), \quad (12)$$

and acts on the plane at an angle $(\theta_1+\theta_2)/2$. The maximum breakdown voltage will be

$$\tau_1 = (p/\pi)\sin\alpha \quad (13)$$

Deformations for points located inside the stretch zone

$$\frac{\partial \overline{u_x}}{\partial x} = -\frac{(1-2\nu)(1+\nu)}{E}p,$$
$$\frac{\partial \overline{u_z}}{\partial x} = -\frac{2(1-\nu^2)}{\pi E}\int_{-a}^{a}\frac{ds}{x-s}, \quad (14)$$
$$\overline{u_x} = -\frac{(1-2\nu)(1+\nu)}{E}px$$

determined by appearance. The deformation for points located inside the $-a \le x \le a$ stretch zone is determined by its appearance.

According to these integrals, the function under the integral has a singularity at $s = x$ and is called a singular integral. In the process of its integration, we divide it into two parts from to and from $s = -a$ to $x - \varepsilon$. Here a is a very small value. Thus, based on the results obtained by Flamand, it is possible to determine the components of the stress tensor corresponding to the deformed state of the half-plane under the action of uniformly distributed vertical forces.

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